Modeling Volatility in Foreign Currency Option Pricing*

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This paper presents a general optimization framework to forecast put and call option prices by exploiting the volatility of the options prices. The approach is flexible in that different objective functions for predicting the underlying volatility can be modified and adapted in the proposed framework. The framework is implemented empirically for four major currencies, including Euro. The forecast performance of this framework is compared with those of the Multiplicative Error Model (MEM) of implied volatility and the GARCH(1,1). The results indicate that the proposed framework is capable of producing reasonable accurate forecasts for put and call prices.(JEL: G12, G13)

Keywords: Foreign currency options, implied volatility, optimal volatility, multiplicative error model, GARCH model

I. Introduction

The well-known Black-Scholes (1973) option pricing model (BS) provides the foundation for pricing of options and derivatives.

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Unfortunately, BS does not evaluate the market's expectation of future volatility, but the expectation can be obtained by inverting the observed option price. For each observed option price, the implied volatility (*IV*) is the volatility implied by the BS option pricing formula given the observed price. This *IV* is widely believed to be the market's best forecast regarding the future volatility over the remaining life of the option. However, *IV* may be a biased representation of market expectations for the following reasons: (i) transaction prices may not represent equilibrium market prices; (ii) the option pricing model may be specified incorrectly; and (iii) as the volatility of asset returns tends to change over time, the constant variance assumption may be unrealistic.

A number of studies have focused on the predictive power of *IV*. The empirical results are at best mixed. Earlier research by Latane and Rendleman (1976), Schmalensee and Trippi (1978), Chiras and Manaster (1978), Beckers (1981) indicate that *IV* is a better predictor of actual volatility than volatility based on historical data. Lamourex and Lastrapes (1993) conduct a joint test of the Hull-White (1987) option pricing model and market efficiency, and they find that although *IV* helps predict volatility, available information in historical data can be used to improve the market's forecasts as measured by *IV*. Day and Lewis (1992) show that *IV* in the equity market contains incremental information relative to the conditional volatility from GARCH models. Similar results are also reported in Fleming et al. (1995), Christensen and Prabhala (1998), Fleming (1998), Bates (2000), and Kazantzis and Tessaromatis (2001). In contrast, Canina and Figlewski (1993) find that *IV* volatility has little predictive power for future volatility. Jorion (1995), however, reports that *IV* outperforms statistical time-series models in terms of information content and predictive power, but *IV* appears to be too variable relative to future volatility.

Harvey and Whaley (1992), using S&P 100 index option, report that implied volatility changes can be predicted ahead of time. This study also indicates that implied volatilities tend to fall on Fridays and rise on Mondays. Using CBOE Market Volatility Index (VIX), an average of S&P 100 option implied volatilities, Fleming et al. (1995), however, reject inter-week seasonality. Furthermore, this study indicates that VIX is inversely related to the contemporaneous S&P 100 index return, and that both daily and weekly VIX changes are more sensitive to the negative than the positive stock market moves. Simon (1997) also reports similar implied volatility asymmetries for treasury bonds and futures options. Ederington and Lee (1996) claim that inter-week patterns of implied volatilities may be attributable to market announcements; they show that the implied volatilities in the treasury bonds and Eurodollar options on futures contracts tend to decline on the days with scheduled macroeconomic announcements.

As widely known, BS is mainly used for valuing options on stocks. This model has spawned the field of financial engineering, which is dedicated to designing and implementing such derivatives pricing models. Has also found wide applications in modeling corporate bonds and credit spreads in the presence of default and interest rate risks (see for a recent application, Belhaj, 2006). For stocks, BS assumes that no dividends are paid on the stock during the life of the option. This model is extended by Merton (1973) for continuous dividends. Since the interest gained on holding a foreign security is equivalent to a continuously paid dividend on a stock share, the Merton version of the BS can be applied to foreign security. To value currency option, stock prices are substituted for exchange rates.

The first application of modern valuation techniques to currency options is generally credited to Grabbe (1983) and Garman and Kohlhagan (1983). They considered foreign currency as an asset and expected returns from holding foreign currency would depend on the volatility of exchange rate in their model. The practical relevance of this model as an approximate currency options pricing formula depends on the investor's ability to forecast exchange rate variability over the remaining life of the option. The model is however, based on several standard assumptions.

This paper provides a new approach to measuring volatility of currency options prices explicitly from their past history. A general optimization framework is proposed to forecast put and call option prices by constructing optimal volatility forecasts based on past information. The volatility is calculated as the weighted sum of the past squared returns by minimizing the in-sample mean squared errors between market and model prices. The future prices are predicted using the BS option pricing model given the volatility forecasts. The objective is to assess how well the past options market prices would forecast the future ones. The emphasis is on assessing the accuracy of the forecasts, rather than on how forecasts are formed.

The paper has several attractive features. First, unlike other approaches in the literature, this paper is concerned with modeling volatility as an instrument to predict future option prices, rather as a measure of risk. Second, this paper proposes a general framework to forecast future option prices. This framework is flexible as it can be modified to accommodate different objective functions to forecast future volatility with different option pricing models. Thirdly, this paper uses the past option prices, rather than the underlying currency prices, to calculate volatility. Although options derive their values from the underlying currencies, spot and options markets are treated as separate entities in this framework. This is a new idea in the options literature. Finally, unlike the majority of work focusing on stocks and bonds options, the current paper focuses on options on major currencies, including Euro.

The paper is organized as follows. The next section gives the analytical framework and the data used in this study, followed by the empirical results in sections III. The last section concludes the paper.

II. Methodology and Data

The framework proposed in this paper can be summarized in the following steps. The first step involves selecting a pricing model to generate future prices. Unless otherwise stated, the pricing model chosen in this paper is the BS option pricing model. Although the constant variance assumption underlying the BS seems restrictive in practice, it is not necessarily the case. It is highly possible that the variance is constant over a short time interval but it is time varying over a longer time horizon. In such a case, the BS is a valid model over each of the short time intervals. This paper assumes that the variance of the underlying asset's return may be constant within a short time interval (one day) but changing from one interval to another, that is, variance changes on the daily basis but constant within the day. The implication of this assumption is that *IV* derived from the BS of a particular day would be a reasonable approximation of the true underlying volatility for that day.

In theory, if *IV* can be predicted ahead of time with reasonable accuracy, then these volatility forecasts can be used as inputs to the BS option pricing formula to forecast future call and put prices. This is the second step of the proposed framework. Given the pricing formula, volatility could be predicted as a weighted average of the past squared returns, with weights calculated by optimizing appropriate objective functions. The above idea can be implemented as a simple spreadsheet-based application, and we name it as optimal weighted volatility (*OV*) model. In this approach, the volatility is modeled as a linear combination of the past squared returns from the observed put and call prices, with weights calculated by minimizing a given objective function. This approach also describes the pattern of the volatility which contains important information about investors' behavior over time. Finally, the forecast performance of BS option pricing formula using *OV* is compared with alternative volatility models including the Multiplicative Error Model (MEM) of *IV* and GARCH(1,1) model of past squared returns. If *OV* provides superior volatility measures for predicting future prices, then this approach will be an innovative way to identify the underlying process of valuing currency options. In what follows, we describe the details of this methodology, and the following notations are used throughout the paper:

S_t spot exchange rate at time t;

T expiration time of the option;

C_t market price of a call option in domestic currency at time t;

P_t market price of a put option in domestic currency at time t;

X_t option exercise price in domestic currency at time t;

$R_t^d \tau$ continuously compounded rate of return on risk-free domestic interest rate with the maturity at time τ;

$R_t^f \tau$ continuously compounded rate of return on risk-free foreign interest rate with the maturity at time τ;

N cumulative normal distribution function;

σ_t volatility of the exchange rate at time t.

Following Biger and Hull (1983), the price of a European call option on currency is,

$$C_t = S_t e^{-R_t^f \tau} N\left(d_{t,1}\right) - X_t e^{-R_t^d \tau} N\left(d_{t,2}\right) = c\left(S_t, X_t, R_t^d, R_t^f, \tau, \sigma_t\right), \quad (1)$$

and similarly, the price of a European put option on currency is stated as,

$$P_t = -S_t e^{-R_t^f \tau} N\left(-d_{1,t}\right) + X_t e^{-R_t^d \tau} N\left(-d_{2,t}\right) \tag{2}$$

$$= p\left(S_t, X_t, R_t^d, R_t^f, \tau, \sigma_t\right),$$

where,

$$d_{1,t} = \frac{\ln\left(S_t / X_t\right) + R_t^d - R_t^f + 0.5\sigma_t^2 \tau}{\sigma_t \sqrt{\tau}},$$

and

$$d_{2,t} = d_{1,t} - \sigma_t \sqrt{\tau}.$$

Equations (1) and (2) are standard, and they state that the option premium is the present value of the difference between two cumulative density functions, d_1 and d_2. These two equations are used in this paper with different volatility measures to generate forecasts for call and put prices. For notation convenience, define

$$\xi_t = S_t e^{-R_t^f \tau}, \quad \eta_t = X_t e^{-R_t^d \tau},$$

so that equations (1) and (2) can be rewritten as

$$C_t = \xi_t N\left(d_{1,t}\right) - \eta_t N\left(d_{2,t}\right), \tag{3}$$

$$P_t = \eta_t N\left(-d_{2,t}\right) - \xi_t N\left(-d_{1,t}\right). \tag{4}$$

Note that in equations (3) and (4), all parameters except the volatility are directly observable from market data. This allows a market-based estimate of volatility of a foreign security. A variety of methods can be applied to estimate the volatility and most researchers use the implied standard deviation (ISD) from option market price as the current estimate of *IV*. Let $v_{P,t}^2$ and $v_{C,t}^2$ denote the *IV* at time *t* for put and call option, respectively, which satisfies the following equations

$$f\left(v_{C,t}\right) = c\left(S_t, X_t, R_t^d, R_t^f, \tau, v_{C,t}\right) - C_t = 0, \tag{5}$$

$$g\left(v_{P,t}\right) = p\left(S_t, X_t, R_t^d, R_t^f, \tau, v_{P,t}\right) - P_t = 0. \tag{6}$$

If put-call parity holds, then $v_{P,t} = v_{C,t} = v_t$. Given both $f\,(v_{C,t})$ and $g\,(v_{P,t})$ are highly non-linear functions, the calculation of *IV* requires numerical procedures such as the Newton-Raphson method. In this paper, a hybrid of Newton-Raphson and Bisection methods are used to calculate *IV*. This procedure is a standard iterative technique based on the first order Taylor expansion of the function.

After the calculation of *IV*, the next step is to forecast future volatility based on this information. One way is to estimate the following model for $v_{i,t}$, $i = C,P$.

$$v_{i,t} = \varepsilon_{i,t} h_{i,t}, \quad \varepsilon_{i,t} \sim iid\left(1,\upsilon\right), \quad \varepsilon_{i,t} > 0 \quad i = C,P.$$

$$h_{i,t}^2 = \omega_i + \alpha_i v_{i,t-1}^2 + \beta_i h_{i,t-1}^2 \quad \omega,\alpha,\beta > 0.$$

This specification implies that the *IV* follows a Multiplicative Error Model (MEM) as proposed in Engle (2002). A sufficient condition to ensure the positivity of $h_{i,t}$ is to restrict all the parameters in the model, namely, ω, α and β to be positive. Given the past information and the parameter estimates, the conditional volatility is estimated by

$$\hat{v}_{i,t} = h_{i,t} \tag{7}$$

for $i = C,P$ and hence, the implied volatility model price (IVP) for calls $\left(C_t^{IV}\right)$ and puts $\left(P_t^{IV}\right)$ can be generated by

$$C_t^{IV} = c\left(S_t, X_t, R_t^d, R_t^f, \tau, \hat{v}_{C,t}\right) \tag{8}$$

$$P_t^{IV} = p\left(S_t, X_t, R_t^d, R_t^f, \tau, \hat{v}_{P,t}\right), \tag{9}$$

It is to be noted that in the MEM presentation as above, the noise sequence $\{\varepsilon_{i,t} : t \in N\}$ is a collection of independently and identically distributed (*iid*) random variables with positive support, that is, $P(\varepsilon_{i,t} \leq 0) = 0$, to ensure that the conditional standard deviation, $h_{i,t}$, to be positive for all *i* and *t*. Given the actual distribution of $\varepsilon_{i,t}$ is generally unknown, this paper estimates the parameters in the MEM using the Quasi-Maximum Likelihood Estimator (QMLE) with log-normal density, that is, the estimates are obtained by maximizing the log-likelihood function with the log-normal density. The statistical properties of this approach can be found in Allen, Chan, McAleer and Peiris (2008).

Another way to forecast volatility is to assume that the return of the prices, $r_{i,t}$ for i = C, P, follow a GARCH(1,1) process. Let C_t^{MP} and P_t^{MP} denote the call and put observed prices, respectively, at time, t. The returns are then calculated as

$$r_{i,t} = \log\left(i_t^{MP}\right) - \log\left(i_{t-1}^{MP}\right), \quad i = C, P,$$

where $r_{i,t}$ is assumed to follow a GARCH(1,1) process, that is,

$$r_{i,t} = \mu_i + \kappa_{i,t} g_{i,t}, \quad \kappa_{i,t} \sim iid\,(0,1) \quad i = C, P$$

$$g_{i,t}^2 = \lambda_i + \gamma_i r_{i,t-1}^2 + \delta_i g_{i,t-1}^2 \quad \lambda, \gamma, \delta > 0.$$

Given this specification, the future GARCH (1,1)-based volatility (GV) for both put and call prices are estimated by

$$\hat{\sigma}_{i,t}^{GV} = g_{i,t} \tag{10}$$

for i = C, P. Although this specification is very similar to the MEM approach as discussed above, the underlying assumptions are quite different. The MEM specification aims to model the volatility and not the return, and consequently, the independent and identically distributed random variable, $\varepsilon_{i,t}$, needs to be positive as mentioned above. In the GARCH(1,1) specification, the endogenous variable is the price return which can be positive or negative and hence, the iid random variable, $\kappa_{i,t}$, can be any real number. However, since $g_{i,t}$ represents the conditional variance of the price return in this case, a sufficient condition to ensure the positivity of $g_{i,t}$ is to restrict all the parameters in the conditional variance equation, namely, λ, $\gamma_{i,}$ δ_i to be positive. Since the distribution of $\kappa_{i,t}$ is generally unknown, the parameters in the GARCH(1,1) model are estimated by QMLE with normal density. The statistical properties of the QMLE for GARCH process can be found in Ling and McAleer (2003).

Using the estimated GV, $\hat{\sigma}_{i,t}^{GV}$, the GARCH (1,1)-based volatility model price (GVP) for calls C_t^{GV} and puts P_t^{GV} can be generated as

$$C_t^{GV} = c\left(S_t, X_t, R_t^d, R_t^f, \tau, \hat{\sigma}_{C,t}^{GV}\right), \tag{11}$$

$$P_t^{GV} = p\left(S_t, X_t, R_t^d, R_t^f, \tau, \hat{\sigma}_{P,t}^{GV}\right) \tag{12}$$

Further to the above two approaches, it is also possible to utilize the past information from both put and call prices directly to forecast the conditional volatility for the purposes of predicting the future prices of both put and call options. We introduce this new idea whereby previous returns of the option prices are utilized and selected as optimal linear combinations of previous absolute returns. The optimal combinations are then used as inputs to the BS option pricing formula as defined in equations (1) and (2) to predict future option prices. For identification purposes, the term *OV* is used to label the conditional volatility obtained in this way. The dynamics of the volatility is assumed to be

$$\hat{\sigma}_t^{OV} = w' r_t \tag{13}$$

where $w' = \left(w_1, ..., w_q\right)'$, $w_i \in [0,1]$ $i = 1..q$, such that, $\sum_{i=1}^{q} w_i = 1$, $r_t = \left(\left|r_{c,t-1}\right|, ..., \left|r_{c,t-1}\right|, \left|r_{p,t-1}\right|, ..., \left|r_{p,t-m}\right|\right)$, so that $l + m = q$. To make equation (13) operational, denote *OV* model prices for put and call by P_t^{OV} and C_t^{OV}, respectively and let P_t and C_t denote the observed prices for put and call options at time *t*, respectively. Note that both P_t^{OV} and C_t^{OV} are functions of $\hat{\sigma}_t^{OV}$, which depends on the weight vector *w*. This *w* vector is obtained by minimizing the with-in-sample mean squared error (MSE) between i_t and i_t^{OV}, $i = C, P$:

$$w = \min_{\omega} T^{-1} \sum_{t=1}^{T} \left[\left(P_t - P_t^{OV}\right)^2 + \left(C_t - C_t^{OV}\right)^2\right]. \tag{14}$$

This is a nonlinear optimization problem and does not have a closed-form solution. It can only be solved by numerical methods. Unless otherwise stated, all the optimal weights presented in this paper were obtained by using the "solver" function in Microsoft Excel®. This procedure has the added advantage that the optimal weights can be calculated without using specialized optimization or statistical software.

Apart from MSE, other objective functions can also be used to select the optimal weight. For example, the mean absolute error (MAE) between i_t^{MP} and i_t^{OV} can be a more appropriate choice if there are excessive amount of outliers and extreme observations in the sample. In that case, the optimal weight is chosen by solving the following minimization problem:

$$w = \min_{\omega} T^{-1} \sum_{t=1}^{T} \left[\left| P_t - P_t^{OV} \right| + \left| C_t - C_t^{OV} \right| \right]. \tag{15}$$

Similarly, the mean absolute percentage error (MAPE) between i_t^{MP} and i_t^{OV} can also be used, in which case the optimal weight is chosen by solving:

$$w = \min_{\omega} T^{-1} \sum_{t=1}^{T} \left[\left| \frac{P_t^{OV} - P_t}{P_t} \right| + \left| \frac{C_t^{OV} - C_t}{C_t} \right| \right]. \tag{16}$$

It is to be noted that the optimal weight vector (w) is assumed to be constant over time.

Next, the optimal-weighted volatility model prices for calls (C_t^{OV}) and puts (P_t^{OV}) are generated by

$$C_t^{OV} = c\left(S_t, X_t, R_t^d, R_t^f, \tau, \hat{\sigma}_t^{OV}\right), \tag{17}$$

$$P_t^{OV} = p\left(S_t, X_t, R_t^d, R_t^f, \tau, \hat{\sigma}_t^{OV}\right), \tag{18}$$

where, $\hat{\sigma}_t^{OV}$, is a function of the past information provided by both call and put prices. In implementing this procedure, our data on calls and puts have the same time to maturity. However, they do not have the same moneyness; when a call is in ITM, the corresponding put is OTM, as the call-put pairs have the same strike prices. Unless otherwise stated, the solutions to the optimization problem as stated in equations (14) to (16) are obtained by using the Solver™ application in Microsoft Excel™ with the default Newton algorithm. Thus, this analysis can be conducted without any additional programming and it would be more suitable as a practical application.

We compute in-sample pricing errors to check for goodness-of-fit, and out-of-sample pricing errors to check for predictive power. Pricing error is defined as the deviation of model price from the observed market price. The forecast performances of the models can be evaluated by the following conventional criteria

The mean squared error (MSE) = $\frac{1}{s}\sum_{t=1}^{s}\left(i_t - i_t^j\right)^2$,

The mean absolute error (MAE) = $\frac{1}{s}\sum_{t=1}^{s}\left|i_t - i_t^j\right|$,

The mean absolute percentage error (MAPE) = $\frac{1}{s}\sum_{t=1}^{s}\left|\frac{i_t^j - i_t}{i_t}\right|$,

for $i = C, P$ and $j = IV, GV, OV$. While MSE is the most commonly used criterion to evaluate forecast errors, it has the well-known problem of penalizing large errors disproportionately. This can result in biases in conclusions if a sample contains excessive amount of outliers and extreme observations. Therefore, the two other alternative criteria can be used to provide a more general view about the forecast performances of each model. The estimated errors are labeled as IVPE, GVPE and OVPE for the three models, respectively. We now proceed to apply the foregoing methodology to the data.

A. The Data

The data used in this paper are for the following four currency options – the British pound, the Euro, the Japanese yen, and the Swiss franc. All data are obtained from DATASTREAM database, and provided in a separate appendix available on request. The data consist of daily closing prices for each option traded on the PHLX, daily spot exchange rates, and daily Eurocurrency interest rates for the period. Option on Euro started trading December 2000. The data set for all currencies, therefore, includes the options trading period from January 2001 to March 2006. There are some inconsistent data (due to recording error in the database) for the Japanese yen from January 2001 to end of March 2001 and consequently, these are excluded from the sample.

The total number of daily observations is 1359 for British Pound, Euro and Swiss Franc and 1300 for Japanese Yen, making a total of 5377 pairs of put-call option prices in our sample. The expiration dates of options are within 90 days during the sample period. If the expiration month has 5 Fridays, the options expire on the third Friday, otherwise second Friday of the expiration month. The Eurocurrency interest rates are used to determine daily domestic and foreign bond prices, respectively.

For a bird’s view, table 1 provides the descriptive statistics of the

TABLE 1. Descriptive Statistics

Currency	Statistics	Call price	Put price	Variables Strike price	Spot rate	Interest rate
British pound	Mean	1.40	1.74	165.06	1.65	4.39
	Median	1.41	1.68	164.00	1.64	4.42
	Skewness	0.01	0.55	–0.02	–0.02	–0.37
	Kurtosis	3.37	3.38	1.57	1.57	2.66
	JB	7.90*	77.04*	115.49*	115.18*	38.02*
Euro	Mean	1.19	1.34	109.71	1.09	3.00
	Median	1.22	1.28	115.00	1.15	2.69
	Skewness	–0.09	3.69	–0.23	–0.23	0.68
	Kurtosis	3.28	37.66	1.58	1.58	2.05
	JB	6.69*	71102.44*	126.47*	126.61*	154.45*
Swiss franc	Mean	1.01	0.83	73.77	0.72	1.55
	Median	0.91	0.81	73.50	0.75	1.22
	Skewness	25.75	0.54	24.66	–0.31	0.83
	Kurtosis	669.29	4.19	631.39	1.81	2.35
	JB	25288077*	147.41*	22497256*	102.17*	181.44*
Japanese yen	Mean	1.03	0.87	86.67	0.01	0.24
	Median	0.98	0.81	85.5	0.01	0.21
	Skewness	1.87	2.78	–0.04	0.03	1.38
	Kurtosis	11.49	26.26	2.12	2.18	5.26
	JB	4876.89*	32393.99*	43.38*	37.94*	719.18*
U.S. dollar	Mean					2.91
	Median					2.88
	Skewness					0.12
	Kurtosis					1.76
	JB					90.66*

Note: The Jarque-Bera (JB) statistic follows a chi-square distribution with 2 degree of freedom. The critical value of the chi-square distribution is 5.99 at the 5% level of significance. The statistical significance level at 5% is denoted by *.

data. As can be seen, for most of the data series, the mean and median values are very close and the skewness is nearly zero. However, the Jarque-Bera (JB) test rejects the null of normality which means the data was unlikely to be drawn from a normal distribution.

III. Empirical Results

In this section we provide the empirical results based on the analytical framework discussed in the previous section. More specifically, to forecast the volatility inputs for call and put options, we estimate (i) option prices based on *IV* model, using equations (8) and (9), (ii) option prices based on return GARCH(1,1) model, using equations (11) and (12), and (iii) option prices based on *OV* model, using equation (13) under alternative weighting schemes. The in-sample results are first discussed, followed by the out-of-sample results.

A. In-sample Fit

For in-sample tests, the implied volatility model pricing error (IVPE) and optimal weighted volatility model pricing error (OVPE) are estimated under the three criteria, namely, MSE, MAE and MAPE. Note that OVPE can be obtained under alternative weighting schemes [see equations (14)-(16)]. The results with MSE as the objective function (for weights), are given in table 2. As can be seen, based on the MSE criterion, the *OV* model outperforms the *IV* model option prices for all four currencies. Under MSE, OVPE is less than IVPE, on average, by 70.55 percent for British pound, 52.91 percent for Euro, 70.95 percent for Japanese yen and 67.23 percent for Swiss franc. It indicates that *OV* model prices fit the in-sample market prices better than those from the *IV* model. However, the MAE and MAPE results in table 2 are not favorable to *OV* model. These two measures indicate that *IV* model tends to do better than the *OV* model. Interestingly, very similar results were observed when MAE and MAPE were used as objective functions, respectively (results not reported here, for brevity).

As the OVPE results in table 2 are based on weights that capture random information from past options prices, we now explore the nature of these weights for each currency over the previous five trading days. The observed weights, under MSE as the objective function, are given in table 3. As can be seen from the last column of the table, over a five-day window, the weights tend to be somewhat evenly distributed across the previous five trading days. However, total weights of call prices are higher than the total weights of put prices for British pound and Swiss franc. For Euro, the total weights of put prices are higher than the total weights of call prices, and somewhat evenly distributed for Japanese yen between calls and puts. Thus, these weights do not seem to follow any systematic pattern across currencies. However, it is to be noted that the options on all four sample currencies are traded against the U.S. dollar in the U.S. market. Since the trading volume influences volatility, the relatively higher weights of call price volatility may indicate that trading volume of call options on British pound and Swiss

TABLE 2. Comparison of OVPE and IVPE: In-Sample

Measures	Currency	Options	OVPE	IVPE	$\frac{\text{OVPE–IVPE}}{\text{IVPE}}$ %	Average difference %
MSE	British	Call	0.0804	0.2322	–65.37	–70.55
	pound	Put	0.1050	0.4327	–75.73	
	Euro	Call	0.0813	0.1655	–50.88	–52.91
		Put	0.1107	0.2457	–54.95	
	Japanese	Call	0.0396	0.1485	–73.33	–70.95
	yen	Put	0.0314	0.0999	–68.57	
	Swiss	Call	0.0368	0.0979	–62.41	–67.23
	franc	Put	0.0298	0.1066	–72.05	
MAE	British	Call	0.6977	0.3556	96.20	84.16
	pound	Put	0.7916	0.4599	72.12	
	Euro	Call	0.6639	0.3267	103.21	100.26
		Put	0.7452	0.3777	97.30	
	Japanese	Call	0.4585	0.3572	28.36	39.23
	yen	Put	0.4473	0.2980	50.10	
	Swiss	Call	0.4345	0.2848	52.56	67.56
	franc	Put	0.4283	0.2346	82.57	
MAPE	British	Call	0.6149	0.2639	133.00	110.44
	pound	Put	0.4864	0.2589	87.87	
	Euro	Call	0.7082	0.3291	115.19	117.12
		Put	0.6350	0.2899	119.04	
	Japanese	Call	0.4436	0.3975	11.60	27.19
	yen	Put	0.4943	0.3462	42.78	
	Swiss	Call	0.5224	0.3265	60.00	65.67
	franc	Put	0.5087	0.2969	71.34	

Note: The columns OVPE through Average difference are grouped under "Model pricing errors and their difference in percentage".

Note: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

franc has been higher than that of put options. This might imply that the U.S. market is a net importer in British pound and Swiss franc denominated goods and services over the sample period. Similarly, higher weights of put price volatility may indicate that the U.S. market is a net exporter in Euro denominated goods and services. With MAE and MAPE as objective functions, very similar results were observed (results not reported here), implying that the weight function does not seem to be sensitive to the choice of the objective function.

TABLE 3. Weights Vector for Full Sample

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t\text{-}1}$	$w_{i,t\text{-}2}$	$w_{i,t\text{-}3}$	$w_{i,t\text{-}4}$	$w_{i,t\text{-}5}$	
British pound	Call	0.1094	0.1258	0.1182	0.1275	0.1462	0.6271
	Put	0.0862	0.0639	0.0665	0.0724	0.0839	0.3729
		0.1956	0.1897	0.1847	0.1999	0.2301	1.0000
Euro	Call	0.0514	0.0030	0.0346	0.0232	0.0425	0.1548
	Put	0.1601	0.1687	0.1732	0.1651	0.1782	0.8452
		0.2114	0.1717	0.2078	0.1883	0.2207	1.0000
Japanese yen	Call	0.1216	0.1024	0.0947	0.1033	0.0882	0.5102
	Put	0.1000	0.0873	0.0848	0.0783	0.1393	0.4898
		0.2216	0.1897	0.1795	0.1816	0.2275	1.0000
Swiss franc	Call	0.1353	0.1100	0.1113	0.1159	0.1420	0.6144
	Put	0.0738	0.0789	0.0697	0.0771	0.0862	0.3856
		0.2091	0.1888	0.1809	0.1930	0.2281	1.0000

Note: $w_{i,t\text{-}1}$, $w_{i,t\text{-}2}$, $w_{i,t\text{-}3}$, $w_{i,t\text{-}4}$ and $w_{i,t\text{-}5}$ represent weights corresponding to the previous day 1, day 2, day 3, day 4 and day 5, respectively.

B. Out-of-sample Fit

In-sample results, in general, indicate that *OV* model outperforms *IV* model for pricing options under the MSE criterion. One may, however, argue that the *OV* model prices fit in-sample better due to the additional explanatory power from higher degrees of freedom. As a check, the out-of-sample predictive power of the *OV* model is now examined in what follows. For this purpose, the predictive power of *OV* model is assessed against *GV* model, and the *IV* model under MEM.

To test the out-of-sample fit of the *OV* model, the weights (reported in table 3) need to be recalculated by using equations (14). Using the first 1000 observations, the estimated weights under MSE as objective function are presented in table 4. As can be seen, the weighing patterns are qualitatively similar to those in the table 3 for the full sample. Overall, the weights estimated from the first 1000 observations to forecast options prices for out-of-sample test are fairly similar to the weights estimated from the full sample. This also holds when MAE and MAPE were used as objective functions (the estimates are available upon requests).

Next, using these new weights, the *OV* model price volatilities are recalculated for the first 1000 observations, which are then used to

TABLE 4. Weights Vector for First 1000 Observations

Currency	Options (i)	Weights corresponding to previous day 1 to day 5					Total Weight
		$w_{i,t\text{-}1}$	$w_{i,t\text{-}2}$	$w_{i,t\text{-}3}$	$w_{i,t\text{-}4}$	$w_{i,t\text{-}5}$	
British	Call	0.1312	0.1394	0.1262	0.1388	0.1592	0.6949
pound	Put	0.0748	0.0501	0.0523	0.0566	0.0714	0.3051
		0.2060	0.1895	0.1785	0.1954	0.2306	1.0000
Euro	Call	0.0378	0.0000	0.0213	0.0069	0.0420	0.1081
	Put	0.1795	0.1673	0.1903	0.1714	0.1833	0.8919
		0.2173	0.1673	0.2116	0.1783	0.2254	1.0000
Japanese	Call	0.1072	0.0796	0.0714	0.0743	0.0648	0.3972
yen	Put	0.1087	0.1020	0.1015	0.1140	0.1765	0.6028
		0.2159	0.1815	0.1729	0.1883	0.2413	1.0000
Swiss	Call	0.1419	0.1139	0.1094	0.1148	0.1470	0.6269
franc	Put	0.0707	0.0765	0.0616	0.0768	0.0874	0.3731
		0.2126	0.1904	0.1710	0.1916	0.2344	1.0000

Note: $w_{i,t\text{-}1}$, $w_{i,t\text{-}2}$, $w_{i,t\text{-}3}$, $w_{i,t\text{-}4}$ and $w_{i,t\text{-}5}$ represent weights corresponding to the previous day 1, day 2, day 3, day 4 and day 5, respectively.

generate the forecast values for the remainder of the sample under this model. Similarly, for *IV* model (under MEM) and *GV* model, volatility points are recalculated for the first 1000 observations, which are then used to generate the forecast values for the remainder of the sample under these models. This implies s = 357 for British Pound, Euro and Swiss Franc and s = 300 for Japanese Yen for purposes of calculating the forecast criteria.

The estimated values of OVPE, under MSE as the objective function are first compared with GVPE, and the results are given in table 5. As can be seen in the last column, the values of OVPE are systematically and considerably smaller than those of GVPE by all measures (MSE, MAE and MAPE). It indicates that the *OV* model performs better than the *GV* model in forecasting option price volatility for all four currencies. Similar results were observed with MAE and MAPE used as objective functions (not reported here).

The out-of-sample performance of *OV* model is then compared with that of *IV* model under MEM. Table 6 gives the results with MSE as the objective function. As can be seen, OVPE does extremely well compared to IVPE with MSE as the test criterion, but the results are somewhat mixed for MAE and MAPE. Results with MAE and MAPE

TABLE 5. Comparison of OVPE and GVPE: Out-of-Sample

Measures	Currency	Options	OVPE	GVPE	$\frac{\text{OVPE–GVPE}}{\text{GVPE}}\%$	Average difference %
			Model pricing errors and their difference in percentage			
MSE	British	Call	0.0631	0.1371	–53.98	–54.62
	pound	Put	0.0711	0.1589	–55.25	
	Euro	Call	0.0520	0.2239	–76.78	–72.30
		Put	0.0425	0.1321	–67.83	
	Japanese	Call	0.0252	0.0398	–36.68	–26.31
	yen	Put	0.0190	0.0226	–15.93	
	Swiss	Call	0.0213	0.0240	–11.25	–16.76
	franc	Put	0.0192	0.0247	–22.27	
MAE	British	Call	0.6407	3.4086	–81.20	–59.24
	pound	Put	0.6889	1.0983	–37.28	
	Euro	Call	0.5833	1.3180	–55.74	–50.48
		Put	0.5323	0.9717	–45.22	
	Japanese	Call	0.3998	0.5084	–21.36	–4.43
	yen	Put	0.3215	0.2858	12.49	
	Swiss	Call	0.3713	0.3957	–6.17	–9.06
	franc	Put	0.3773	0.4285	–11.95	
MAPE	British	Call	0.4388	0.7263	–39.58	–40.05
	pound	Put	0.4040	0.6791	–40.51	
	Euro	Call	0.4779	1.1239	–57.48	–52.59
		Put	0.4741	0.9064	–47.69	
	Japanese	Call	0.4137	0.5434	–23.87	–8.52
	yen	Put	0.4477	0.4191	6.82	
	Swiss	Call	0.3824	0.4265	–10.34	–13.75
	franc	Put	0.4783	0.5774	–17.16	

Note: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than GVPE and OVPE is more than GVPE, respectively, by reported percent.

used as objective functions (not reported here) were also very similar. Overall, *OV* model tends to outperform the *GV* and *IV* models.

IV. Conclusion

This paper introduces a new approach to computing the volatility explicitly from the currency options market prices. The idea is to assess

TABLE 6. Comparison of OVPE and IVPE: Out-of-Sample

Measures	Currency	Options	OVPE	IVPE	$\frac{\text{OVPE–IVPE}}{\text{IVPE}}$ %	Average difference %
			Model pricing errors and their difference in percentage			
MSE	British	Call	0.0631	0.4303	–85.34	–85.25
	pound	Put	0.0711	0.4793	–85.17	
	Euro	Call	0.0520	0.2689	–80.66	–80.49
		Put	0.0425	0.2159	–80.31	
	Japanese	Call	0.0252	0.2238	–88.74	–83.11
	yen	Put	0.0190	0.0844	–77.49	
	Swiss	Call	0.0213	0.1598	–86.67	–84.80
	franc	Put	0.0192	0.1125	–82.93	
MAE	British	Call	0.6407	0.6108	4.90	2.07
	pound	Put	0.6889	0.6941	–0.75	
	Euro	Call	0.5833	0.5569	4.74	6.84
		Put	0.5323	0.4886	8.94	
	Japanese	Call	0.3998	0.5970	–33.03	–9.91
	yen	Put	0.3215	0.2840	13.20	
	Swiss	Call	0.3713	0.3779	–1.75	9.00
	franc	Put	0.3773	0.3151	19.74	
MAPE	British	Call	0.4388	0.3959	10.84	6.08
	pound	Put	0.4040	0.3987	1.33	
	Euro	Call	0.4779	0.4445	7.51	7.98
		Put	0.4741	0.4372	8.44	
	Japanese	Call	0.4137	0.5982	–30.84	–9.14
	yen	Put	0.4477	0.3977	12.57	
	Swiss	Call	0.3824	0.3865	–1.06	11.15
	franc	Put	0.4783	0.3877	23.37	

Note: OVPE and IVPE represent optimal weighted volatility model pricing error and implied volatility model pricing error, respectively. MSE and MAE measures are to be divided by 1000 and 100, respectively. In the last column, the average negative and positive differences indicate that OVPE is less than IVPE and OVPE is more than IVPE, respectively, by reported percent.

how well the past options market prices would forecast the future ones, thereby the focus being on the accuracy of the forecasts, rather than on how forecasts are formed. In this framework (*OV* model), a process of an optimal linear combination of past absolute returns is generated by minimizing different objectives functions (MSE, MAE, MAPE). The forecast performance of *OV* model is then compared to that of Engle's (2002) multiplicative error model for *IV* and a GARCH (1,1) model. Overall, the results indicate that the proposed *OV* model in this paper is capable of producing reasonably accurate forecasts for the put and call prices. The empirical results of this paper have important implications for option traders who need to use forecasting model for options valuation purposes. This paper has provided a general framework that is computationally less burdensome, and can be easily implemented in spreadsheet applications. More accurate formulae would require solving quadratic or higher order algebra equations, for which no simple closed-form solutions can be obtained. The model proposed in this paper is simple and robust relative to MEM and GARCH(1,1) for forecasting option prices. This model is also flexible as it can be modified to accommodate different objective functions to forecast future volatility with different option pricing models. In future research, it will be interesting to compare this approach with other stochastic models where the implied volatility is updated daily. A related interesting area will be to compare this approach with the analytical approximations proposed by Datey (2003) for the computation of Asian quanto-basket-type option prices. A good estimation of future volatility surface across strike prices is also another possible area of future research (see, for example, Klebaner, Le and Lipster, 2006).

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