Comparing Conditional Variance Models: Theory and Empirical Evidence

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The aim of this paper is to identify whether the GARCH or the SV based models provide the best goodness of fit to financial time-series data. To investigate the issue, three different formulations for each type (i.e., the standard model, the fat-tailed model, and the asymmetric model) are examined. The models are first compared on theoretical grounds, then estimated using the daily returns from four market indices, and finally subjected to some diagnostic tests. The results demonstrate that for the standard formulation, the SV model fits data better than the GARCH model, while the fat-tailed and the asymmetric models roughly equivalent in describing the key features of returns. The results provide a preliminary analysis for selecting the best model with which to forecast the volatility of financial returns (JEL G0,G1).

Keywords: GARCH models, stochastic volatility models, QML estimation, financial time series.

I. Introduction

In order to describe the stylized facts which typically characterize the time series of financial returns, such as time-changing variance, clustering, persistence, leverage effect, and strong autocorrelations in the squared returns, two different classes of parametric models, known

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as conditional variance models, are generally applied and studied in the literature: the generalized autoregressive conditionally heteroskedastic (GARCH) models (Engle [1982], Bollerslev [1986]) and the stochastic variance or stochastic volatility (SV) models (Taylor [1986]).

Since Engle's seminal paper (1982) proposing an ARCH model on the basis of the fundamental distinction between conditional and unconditional variance, financial econometrics literature in the field of conditional variance models has witnessed an extraordinary growth, both in theory and in applications, with the addition of new and more powerful extensions, e.g., the GARCH model by Bollerslev (1986) and the EGARCH model by Nelson (1991), which have dramatically improved the fitting performances of the initial ARCH formulation.

Taylor (1986), on the other hand, offers an alternative with which to model the conditional variance of financial returns, the stochastic variance model. Together with its extensions (Harvey et al. [1994], Harvey and Shephard [1996], Sandmann and Koopman [1998], Jacquier et al. [2001], and Chib et al. [2001]), the proposed SV model captures more qualitative features of financial returns. In comparison with ARCH-GARCH models, which consider conditional variances as a deterministic function of past returns, the SV models describe variance (or rather the log-conditional variance) as a stochastic latent process, which however can be estimated. Besides, ARCH-GARCH can be easily estimated by the Maximum Likelihood and Quasi Maximum Likelihood techniques, while, in the SV framework, the presence of the latent variable makes estimation a harder task.

These difficulties however have boosted research into estimation techniques, and now it is possible to choose among different estimation methodologies: Generalized Method of Moments (Andersen and Sørensen [1996]), which is an evolution of the earlier Moments Matching applied by Taylor (1986; 1994) and Quasi Maximum Likelihood (Ruiz [1994], Harvey et al. [1994] and Harvey and Shephard [1996]). In addition, a number of simulation methodologies can be used: Importance Sampling (Danielsson and Richard [1993]), Indirect Inference (Gouriéroux et al. [1993]), Efficient Method of Moments (Gallant and Tauchen [1996]), Monte Carlo Likelihood (Durbin and Koopman [1997], Sandmann and Koopman [1998]), and Monte Carlo Markov Chain (Jacquier et al. [1994 and 2001], Kim et al. [1998] and Chib et al. [2001]). The first group of techniques for estimating SV models (GMM and QML) are easy to apply and less time-consuming, while the latter (IS, II, EMM, MCL, MCMC) sometimes appear to be more efficient but are often extremely time-consuming.

The need for conditional variance models to help to ensure the best fit to empirical data is due to the ever-increasing importance which variance plays in modern financial theory and applications, in particular in option pricing, risk management, the construction of optimal assets portfolios and other issues.

After analyzing and comparing at a theoretical level the two classes of models, ARCH-GARCH and SV, they are fitted to the data and the empirical results are compared, to contribute to the identification of the best data-fitting model among the various ARCH-GARCH and SV formulations considered here. In particular, for each class, three formulations are chosen: a standard version, a fat-tailed and an asymmetric one. For GARCH, this turns out to be the comparison of GARCH(1,1) (Bollerslev [1986]), GARCH(1,1)-t (Bollerslev [1987]) and EGARCH(1,1) (Nelson [1991]) models. For the SV models, the SV standard (Taylor [1986]), the SV-t (Harvey et al. [1994]) is compared to the asymmetric SV (Harvey and Shephard [1996]). For the empirical strategy to verify goodness of fit some diagnostic tests are carried out for each class of models and between the two classes.

This paper is organized as follows: Section II examines the theoretical underpinnings and estimation of the GARCH and SV type models. Section III discusses the application of the aforementioned models using financial time series data. Section IV presents the summary of the results and conclusions.

II. Some Theoretical Aspects

A. Introduction to Conditional Variance Models

The econometric models most frequently applied to describe the stylized facts of financial returns are based on the following general equation (Taylor [1986], Harvey and Shephard [1993], Bollerslev et al. [1994], Ghysels et al. [1996], Campbell et al. [1997])

$$r_t = \mu_t + \xi_t \sigma_t \tag{1}$$

where r_t stands for the return at time t, ξ_t is an IID random variable with zero mean and unitary variance, and σ_t is volatility at time. The variable $\mu_t = \mu(I_t)$ is a function of I_t, the information set available at time t

(Andersen [1994]). It represents the trend (Ghysels et al. [1996]) or rather, the mean or the level of the returns data generating process (DGP). Usually, when returns are not correlated, μ_t is simply the sample mean of the returns. In order to represent a possible linear structure, the stochastic processes most frequently used are the ARMA (Box and Jenkins [1976]). The mean μ_t can also include exogenous variables (Bollerslev [1986], Harvey and Shephard [1993], Sandmann and Koopman [1998), such as seasonal and days dummies or trading volumes (Tauchen et al. [1996], Lamourex and Lastrapes [1994]). By subtracting μ_t from r_t it follows that

$$y_t = \xi_t \sigma_t \tag{2}$$

where $y_t = r_t - \mu_t$. The main assumption in equation (2) is that y_t is a white noise, and for this reason y_t is defined as the pre-whitened returns series. With reference to equation (1), there are two possible sources of variability in the model: the mean μ_t and volatility σ_t. Usually, the mean is smaller in magnitude than volatility, suggesting that most of the variability is due to the latter.

Let us now consider a non-linear transformation (call it f) of volatility σ_t, which is represented as a function of past information I_t,

$$f\left(\sigma_t\right) = g\left(I_t\right). \tag{3}$$

Conditional on I_t, the main assumption is that y_t is distributed as normal,

$$y_t \mid I \sim N\left(0, \sigma^2\right). \tag{4}$$

Given the general model described by equations (1) and (3) and assumption (4), two different types of models can recognized, related to the different way in which the functions f and g are specified and parameterized (Shephard [1996]):

Observation-driven models, also defined by Taylor (1986) as models in which the variance changes are caused by past prices, where $f(\sigma_t)$ is a function of the realized returns until t.

Parameter-driven models, otherwise defined as models in which the variance changes are not caused by prices (Taylor [1986]), where

$f(\sigma_t)$ depends on a latent or unobserved variable.

In the first class are the generalized autoregressive heteroskedasticity models (GARCH and variants), while more frequently used among the second class are the Stochastic Volatility (SV) models.

For the GARCH models the function f is usually specified as the square of volatility σ_t, g is structured as an ARMA process and includes past squared returns and possibly past conditional variances (or log-variances for the EGARCH). In this case, $y_t \mid (I_t = Y_t)$ is distributed as $N(0,\sigma^2)$ (Engle [1982], Bollerslev [1986], Nelson [1991]), where I_t is composed of the set of returns observed until time t–1.

The SV models $f = \ln(\sigma_t^2)$ is the logarithm of the conditional variance,(as in the EGARCH specification) and g is usually specified as the sum of an AR(1) process and a random error. Thus, the SV model considers volatility as a stochastic variable. The conditional distribution of y_t is then the following (Taylor [1994]):

$$y_t \mid \left(I_t = h_t\right) \sim N\left(0, \exp\left(h_t\right)\right),$$

where I_t is log-volatility, $h_t = \sigma_t^2$, regarded as a latent variable which can be estimated on the basis of the observations on returns.

B. GARCH Models

Referring to equation (2), in the GARCH(p,q) model the conditional variance is characterized by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{5}$$

where p refers to the lags of the variable σ_t^2 and q to the lags of the variable y_t. When p=1 and q =1, the GARCH(1,1) model is obtained,

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{6}$$

The GARCH model may be generalized by letting ξ_t in equation (2) have a student t distribution with v degrees of freedom. The importance of this class of models, known as GARCH- t (Bollerslev [1987]), is due to the fact that they are able to capture the excess kurtosis present in many financial time series.

The EGARCH(p,q) model, on the other hand, considers the logarithm of the conditional variance as follows

$$\ln\left(\sigma_t^2\right) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(\left|\xi_{t-i}\right| - E\left(\left|\xi_{t-i}\right| + \gamma \xi_{t-i}\right)\right) + \sum_{j=1}^{p} \beta_j \ln\left(\sigma_{t-j}^2\right) \quad (7)$$

where ξ_t are the standardized residuals. This model allows us to describe the asymmetric effect which is often present in the returns series. As explained by Nelson (1991), if $\alpha_i > 0$ a deviation of ξ_t from its expected value causes the conditional variance of y_t, σ_t^2, to be bigger than it otherwise would be. The parameter γ allows this effect to be asymmetric: if $\gamma = 0$, a positive ξ_t will have the same effect on volatilities produced by a negative ξ_t of the same magnitude. But if $-1 < \gamma < 0$, a positive ξ_t will increase future volatilities less than a negative ξ_t. Finally, if $\gamma < -1$, a positive ξ_t will reduce future volatilities while a negative one will produce the opposite effect.

C. Stochastic Volatility Models

The basic stochastic volatility model is

$$y_t = e^{h_t/2} \xi_t \quad (8)$$

$$h_t = \alpha + \phi h_{t-1} + \eta_t \quad (9)$$

where $\eta_t \sim NID(0, \sigma_\eta^2)$, $h_t = \ln(\sigma_t^2)$ is the volatility function and working with logarithms ensures that σ_t^2 is always positive; ξ_t is a white noise process with unit variance generated independently of η_t and $|\phi| < 1$, which ensures the strict stationarity of the process. It follows that h_t is stationary with mean $\alpha_h = \alpha/(1-\phi)$ and variance $\sigma_h^2 = \sigma_\eta^2/(1-\phi^2)$.

Despite a very parsimonious representation, this model captures most of the empirical regularities found in financial time series (Ghysels et al. [1996]). The component σ_t is known in financial literature as volatility or conditional variance (see, for istance, Engle [1982], Bollerslev [1986], Taylor [1994]). In particular, one interpretation of the process h_t, which has its origin in Clark (1973) and is refined in Tauchen and Pitts (1983), is that stochastic volatility reflects the random and uneven flow of new information to the financial markets.

Transforming y_t of equation (8) by taking logarithms of the squares, the following measurement equation of a linear state space model is obtained

$$\ln y_t^2 = E\left(\ln \xi_t^2\right) + h_t + \varepsilon_t \,, \tag{10}$$

where $\varepsilon_t = \ln(\xi_t^2) - E(\ln(\xi_t^2))$. If ξ_t is distributed as a standard normal, $\ln(\xi_t)$ follows a non-Gaussian distribution with mean $\psi(1/2) - \ln(½) \approx -1.27$ and variance $\pi^2/2$ (Ruiz [1994], Sandmann and Koopman [1998]) where $\psi(\cdot)$ is the digamma function (Abramovitz and Stegun [1970]).

Equation (8) may be written as

$$y_t = \beta e^{h_t/2} \xi_t \tag{11}$$

where β is a scale parameter that allows the state space model to be rewritten as

$$\ln y_t^2 = \omega + h_t + \varepsilon_t \tag{12a}$$

$$h_t = \phi h_{t-1} + \eta_t \tag{12b}$$

where $\omega = \ln(\beta^2) + E(\ln(\xi_t))$. As $\ln(\beta^2) = \alpha/(1-\phi)$, if the ϕ and β parameters are known, it is easy to obtain the value of α.

When ξ_t has a student t distribution with v degrees of freedom, equation (8) represent a SV-t model. It can be shown (see, Harvey et al. 1994) that when h_t is stationary, y_t is white noise and from the properties of the t distribution it follows that the unconditional variance generalizes to

$$\left(\frac{v}{v-2}\right) e^{\alpha_h + \sigma_h^2/2}.$$

In this case ξ_t of the equation (8) may be written

$$\xi_t = \frac{\zeta_t}{\sqrt{\kappa_t}}, \tag{14}$$

for $t = 1, \ldots, T$, where ζ_t is a standard normal variate and $v\ \kappa_t$ is distributed, independently of ζ_t as a χ^2 with v degrees of freedom. Thus

$$\ln\left(\xi_t^2\right) = \ln\left(\zeta_t^2\right) - \ln\left(\kappa_t\right) \tag{15}$$

and it follows that the mean and variance of $\ln(\kappa_t)$ are respectively $\psi'(v/2) - \ln(v/2)$ and $\psi'(v/2)$, where $\psi'(\cdot)$ is the trigamma function (Abramovitz and Stegun [1970]).

When there is a dependence between ξ_t and η_t the model picks up the kind of asymmetric behavior that is often found in stock prices (Schwert [1989], Nelson [1991], Engle and Ng [1993], Nicolis [2000]). The asymmetry refers to the impact of negative returns on predicted volatility with respect to the impact of positive ones. For example, a negative return tends to be associated with an increase in predicted volatility, suggesting a negative correlation between ξ_t and η_t of equations (8) and (9).

The linear state space form (13) can be modified to estimate asymmetric models. Harvey and Shephard (1996) observed that, even if ξ_t and η_t in the equation (8), respectively, and equation (9) are correlated, the disturbances in the linear state space form are uncorrelated provided the joint distribution of ξ_t and η_t is symmetric. Hence, as in the state-space transformation, taking the square of the observations, the information on the dependence between ξ_t and η_t is lost. Harvey and Shephard (1996) showed that this information can be recovered by conditioning s_t, which is the sign of the observations, which of course is the same as ξ_t. The linear state space form including the asymmetric component becomes the following

$$\ln y_t^2 = \omega + h_t + \varepsilon_t \tag{16}$$

$$h_{t+1} = \phi h_t = s_t \mu^* + \eta_t^* \tag{17}$$

$$\begin{bmatrix} \varepsilon_t \\ \eta_t^* \end{bmatrix} | \, s_t \sim IID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \gamma^* s_t \\ \gamma^* s_t & \sigma_\eta^2 - \mu^{*2} \end{bmatrix} \right) \tag{18}$$

where μ^* denotes the expectation taken conditional on ξ_t being positive, $\mu^* = E_+(\eta_t)$, and γ^* assigns a similar interpretation to variance and covariance operators, $\gamma^* = \text{cov}(\eta_t, \varepsilon_t)$.

D. Estimation

It is very quick and easy to estimate GARCH models through maximum likelihood or quasi maximum likelihood via iterative numerical algorithms (see, Engle [1982], Bollerslev [1986]). On the contrary, the estimation procedure of SV is a very difficult task because the vector of latent observations $\{h_t\}$, for $t = 1, 2, ..., T$, has to be integrated out of the joint density of the volatility and returns in one T-dimensional integration,

$$\ell(\Omega) = \int_{R^T} f(y, h \mid \Omega)\, dh \qquad (19)$$

to obtain the likelihood function $\ell(\Omega)$, where Ω is the vector of the parameters $(\alpha, \sigma_\eta, \phi)$. This integral does not have an analytical solution. So numerical methods (which are often very computer-intensive and quite difficult to implement) must be employed for evaluation. This is mainly what renders the SV formulation less attractive than the GARCH one, and explains why the latter has been more widely adopted. On the other hand, the computational problems it generates have created a challenge for researchers to develop more efficient and computer-intensive estimation methods such as QML, GMM, EMM and MCMC (for a survey on a comparison of estimation techniques, see Ghysels et al. [1996], Shephard [1996], Jacquier et al., [1994], Sandmann and Koopman [1998] and Andersen et al. [1999]). In this work, the QML approach based on the Kalman filter is considered for estimating the different types of SV models (Harvey et al. [1994], Ruiz [1994], Harvey and Shephard [1996]). The chosen estimation method represents a good compromise between efficiency and computational time consumption.

Considering the equation (8) when $\xi_t \sim NID(0,1)$ if y_t is squared and logged, the model can be rewritten as

$$\ln y_t^2 = -1.27 + h_t + \varepsilon_t \qquad (20a)$$

$$h_t = \alpha + \phi h_{t-1} + \eta_t \qquad (20b)$$

where $\varepsilon_t = \ln(\xi_t^2) - E(\ln(\xi_t^2))$ follows the χ_1^2 distribution with mean zero and variance $\pi^2/2$. The QML method approximates the distribution of ε_t by $NID(0, \pi^2/2)$. The system (20) represents the measurement and transition equations of the general linear state model in which the state space form relates an observed time series y_t to an unobserved state vector h_t. Using Kalman filter the Gaussian likelihood by the prediction error decomposition is obtained. The parameter estimates of $\omega = (\alpha, \phi, \sigma_\eta^2)$ result from numerical optimization.

The standard theory for the estimation of unobserved component time series models with non-normal errors applies to the estimate of ω (Ruiz [1994]).

When ξ_t in equation (8) is a student t variable with v degrees of freedom as given by (14), the variance of of the model (20) is $\sigma_\varepsilon^2 = \pi^2/2 + \psi'(v/2)$. The estimated parameter set becomes $\omega^* = (\alpha, \phi, \sigma_\eta^2, \sigma_\varepsilon^2)$ with the restriction $\sigma_\varepsilon^2 \geq \pi^2/2$. The estimates of ω^* can be obtained by the QML procedure approximating the distribution of ε_t by $NID(0,\sigma_\varepsilon^2)$, and maximizing the resulting quasi-likelihood function (see Ruiz [1994], for the asymptotic theory).

Let us consider now the estimation of the asymmetric SV model. Assuming that the joint distribution of ξ_t and η_t is symmetric, the disturbances in the linear state space form of (16) and (17) are uncorrelated. For this, Harvey and Shephard (1996) suggest the "unrestricted" QML method for estimating the parameters ω, ϕ, σ_η^2, σ_ε^2, μ^* and γ^*. The QML estimators are obtained by treating η_t and ε_t as though they were normal and maximizing the prediction-error decomposition form of the likelihood achieved via the Kalman filter. This procedure estimates the parameters without any distributional assumption, apart from the existence of fourth moments of η_t and ξ_t. As a consequence it doesn't provide an estimate of the parameters of the joint distribution of η_t and ξ_t which is denoted by ρ. Such an estimate can be constructed by making a distributional assumption about ξ_t and as well as η_t.

When the joint distribution of η_t and ξ_t is bivariate normal with correlation ρ, $E(\eta_t \mid \xi_t) = \rho\, \sigma_\eta\, \xi_t$, thus

$$\mu^* = \rho \sigma_\eta \sqrt{2/\pi} = 0.7979 \rho \sigma_\eta \,. \tag{21}$$

Moreover,

$$\gamma^* = \rho \sigma_\eta E\left(|\, \xi_t \,| \ln \xi_t\right) - .7979 \rho \sigma_\eta E\left(\ln \xi_t^2\right) = 1.1061 \rho \sigma_\eta \tag{22}$$

(see Harvey and Shephard [1996], for detailed results). As for the standard model, it can be shown that the QML estimator is consistent and asymptotically normal. Even if conditioning on the sign s_t of the observations complicates matters somewhat in that the covariance between the two disturbances in equation (18) varies according to s_t, consistency and asymptotic normality can nonetheless be demonstrated (Harvey and Shephard [1996]).

E. Differences and Analogies

From sub-sections A and C it stands out that the fundamental difference between the GARCH and the SV models consists of a different specification of the equation describing the conditional variance. Other important differences refer to the sources of variability, restrictions on parameters, stationarity conditions, unconditional kurtosis and estimation techniques.

Sources of variability and restrictions on parameters

The GARCH models have only one source of variability which stands for the noise in equation (1), while SV is characterized by two separate sources of variability: the noise ξ_t in equation (8) and innovation η_t in the volatility equation (9). The latter makes the conditional variance a stochastic process and determines the degree of mixing of the returns. In the GARCH class case, in particular when adopting a high order of parameters, increasing problems in managing and interpreting parameter estimates have to be tackled. Besides, it takes a lot of restrictions on parameter values to guarantee that, for each t the conditional variances σ_t^2 are non-negative. In the EGARCH formulation the problem is solved considering the logarithm of the variance. When fitting the SV model, only a few restrictions are necessary to perform estimation, and the parameter meanings are easy to interpret. As in the EGARCH formulation, the SV model describes the logarithm of the variance, and no other parametric bounds are required in order to estimate the model.

Parameterization and Stationarity Conditions

The squared returns of a generic GARCH model can be re-parameterized as an ARMA process (Bollerslev [1986]). The squared returns of the SV model behave like the ARMA (more precisely, an ARMA(1,1)) as well, especially as $\exp(\sigma_t^2)/(1-\phi^2) \rightarrow 0$ and $\phi \rightarrow 1$ (Taylor [1986]). For this reason, the ϕ parameter takes on the same interpretation of $\Sigma\,\alpha_i + \Sigma\,\beta_j$, that is a measure of volatility persistence. Moreover, in order to ensure the (weak) stationarity of the conditional variance (and hence of the returns), the GARCH models require that $\Sigma\,\alpha_i + \Sigma\,\beta_j < 1$ while the SV class needs $|\phi| < 1$.

Unconditional kurtosis

The widely used GARCH specification in the conditional variance models literature, i.e., the GARCH(1,1) with normal conditional distribution, is adequate to describe returns of conditional heteroskedasticity, but it shows major limits when representing the high kurtosis of unconditional distributions of return (Geweke [1994], Shephard [1996], Starica and Pictet [1997], Kim et al. [1998]). Bai et al. (2001) show that y_t's unconditional kurtosis can be broken down into two components: kurtosis induced by the persistence parameters $\alpha_1 + \beta_1$, and kurtosis generated by the distribution of noise ξ_t, the two components acting in a symmetric and interactive way to determine overall kurtosis. In particular, the authors prove that, for the GARCH(1,1) with $\xi_t \sim N(0,\,1)$, no contribution to the unconditional kurtosis arises from the innovation distribution, but that for the values of $\alpha_1 + \beta_1$ more frequently found in empirical applications, that is $0.85 < \alpha_1 + \beta_1 < 1$, the induced kurtosis is too small to replicate the high unconditional kurtosis. Thus Bollerslev (1987), Nelson (1991), Shephard (1996), Terasvirta (1996), Mikosch and Starica (2000), among others, suggest the use of GARCH processes with fat-tailed errors, like *t* student or GED. In the SV case, the kurtosis coefficient of the unconditional returns has the following expression (Taylor [1986], Taylor [1994], Liesenfeld and Jung [2000], Bai et al. [2001])

$$\kappa = E\left(\varepsilon_t^4\right)e^{\sigma_h^2}. \tag{23}$$

Equation (23) shows that the unconditional distribution kurtosis of y_t is the result of two separate components which, as in the GARCH case, operate symmetrically and interactively: conditional kurtosis, depending on the innovation ξ_t distribution, and kurtosis induced by the volatility unconditional variance, $\sigma_h^2 = \sigma_\eta^2/(1-\phi^2)$, which in turn depends on the persistence parameter ϕ. Nevertheless, unlike the GARCH models, if $\xi_t \sim N(0,\,1)$, conditional kurtosis is equal to 3, and thus, since

$e^{\sigma_h^2} > 0, \kappa > 0$, in (23). As a consequence, the SV model can be defined with normal conditional distribution as a thick-tailed model. However, Liesenfeld and Jung (2000) show that for the persistence values most likely in empirical applications, i.e., with $\phi > 0.9$, the normal SV model does not fit well enough to capture the entire unconditional kurtosis, this being consistent with results obtained by Geweke (1994), Terasvirta (1996), Gallant et al. (1997), among others. So, for the SV as well, it is necessary to adopt heavy-tailed conditional distributions. Moreover, in the GARCH models, the fourth moment may not exist, in which case it is not possible to calculate unconditional kurtosis. On the contrary, in the SV models, the fourth moment exists whenever h_t is a stationary process: thus it is always possible to calculate kurtosis.

Conditional Variance Asymmetry

For the EGARCH formulation He et al. (1999) show that, under specific assumptions of the expected values of certain functions of returns innovation, even when $\varepsilon_t \sim N(0, 1)$, unconditional kurtosis is $\kappa > 3$ and depends on parameter values. Although EGARCH returns are more fat-tailed than GARCH ones, the normal distribution can not describe the entire unconditional kurtosis. As for the standard SV formulation, equation (23) is still valid in the asymmetric case.

Estimation Algorithms

The method usually utilized for estimating the conditional variance models is the QML technique. More precisely, for the GARCH model class it easy to maximize the likelihood function via iterative numerical algorithms. Instead, in the SV model class it is not possible to analytically solve the likelihood function which is T-dimensional integrated. For this, as already seen in section II.C, it is necessary to use a more complex procedure based on the Kalman Filter (Harvey and Shephard [1996]).

III. Empirical Application

A. Dataset

To implement empirical comparisons a series of four indices are considered: (1) Deutsche Aktienindex (DAX): 2119 observations, from

TABLE 1. Main Descriptive Statistics

	Mean×10^3	St. dev.×10^3	Skewness	Kurtosis	JB	LB(30)
DAX	0.654	12.962	–0.430	5.677	703	1071
Dow Jones	0.565	9.832	–0.460	8.058	2359	552
FTSE100	0.333	9.713	–0.160	4.389	187	1537
MIB30	0.627	14.986	–0.073	4.415	179	935

TABLE 2. LM Test Statistics and Their Critical Values

Lag	DAX	Dow Jones	FTSE100	MIB30	χ^2
1	139.37	91.374	58.815	100.705	3.841
5	221.207	164.874	219.74	225.011	11.071
30	293.7	228.431	333.178	304.026	43.773
50	322.599	251.802	359.503	325.319	67.505

Note: For the details on LM tests, see Engle (1982). The χ^2 critical values at at the 5%

04/01/1993 to 01/06/2001; (2) Dow Jones (DJ): 2123 observations, from 04/01/1993 to 01/06/2001; (3) Financial Times Stock Exchange (FTSE100): 2124 observations, from 04/01/1993 to 01/06/2001; (4) Milano Indice Borsa (MIB30): 2124 observations, from 04/01/1993 to 01/06/2001.

The choice of these indices is due to the fact that they represent the different stock exchange markets. The returns from the index series collected each trading day at closing time are used. The values are the continuously compounded returns calculated as the natural logarithm of two consecutive index values, $r_t = \ln(P_t/P_{t-1})$, a transformation used to obtain approximately stationary series. Returns are then filtered through suitable ARMA processes to eliminate or reduce any data linear structure. Residuals (or adjusted returns) obtained are approximately distributed as white noise.

Table 1 presents several descriptive statistics of the data. The kurtosis values indicate that the sample distributions of the adjusted returns have heavy tails, especially for Down Jones. In addition, the Jarque-Bera (1987) tests clearly show non-normality of distribution of the four indices. The Ljung-Box (1978) tests (LB test) show strong autocorrelations in the squared returns distributions. The Engle (1982)

TABLE 3. Asymmetry Tests

	Global standard deviation σ	Positive standard deviation σ^+	Negative standard deviation σ^-	Absolute percentage difference $\Delta\%$	Number of positive m	Number of negative n
DAX	0.013	0.0118	0.0142	20.34	1160	957
Dow Jones	0.0098	0.0091	0.0106	16.48	1141	978
FTSE100	0.0097	0.0093	0.0102	9.68	1109	1011
MIB30	0.015	0.0148	0.0153	3.38	1080	1035

Note: For the details on the tests see Drobetz and Zimmermann (2003).

LM test in table 2 indicates the presence of ARCH effects in the conditional variance at 1, 5, 30 and 50 lags.

Table 3 shows the results of the procedure suggested by Drobetz e Zimmermann (2003) for detecting the presence of asymmetry in financial returns series and performed before fitting any model to the data. In short, the technique requires the splitting of each dataset into two separate subsets, the positive returns subset (r_t^+) and the negative returns subset (r_t^-). Applying the following equations, the standard deviation for each subset minus the whole dataset mean value is calculated:

$$\sigma^+ = \sqrt{\frac{1}{m-1}\sum_{t=1}^{m}\left(r_t^+ - \frac{1}{T}\sum_{t=1}^{T} r_t\right)^2} \tag{24a}$$

$$\sigma^- = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}\left(r_t^- - \frac{1}{T}\sum_{t=1}^{T} r_t\right)^2} \tag{24b}$$

where m and n represent the number of observations for each subset. The quantity $\Delta\%$ indicates the absolute percentage difference between σ^+ and σ^-.

If the positive subset standard deviation is smaller (bigger) than the negative (positive) subset, asymmetry is likely to be present in the adjusted returns datasets, i.e. bad news and good news with the same absolute value have a different impact on future volatility (Bali [2000], Engle and Ng [1993]). It is possible to briefly conclude that the four

TABLE 4. GARCH(1,1) Estimates

Estimates	DAX	Dow Jones	FTSE100	MIB30
α_0	2.72×10^{-4}	8.16×10^{-7}	4.51×10^{-7}	1.32×10^{-5}
	(4.96×10^{-5})	(2.05×10^{-7})	(1.71×10^{-7})	(3.01×10^{-6})
	[5.48]	[4.20]	[2.64]	[4.38]
α_1	0.089	0.083	0.045	0.117
	(0.010)	(0.007)	(0.007)	(0.017)
	[8.55]	[12.07]	[6.61]	[7.08]
β_1	0.895	0.912	0.951	0.824
	(0.011)	(0.007)	(0.007)	(0.025)
	[77.9]	[118.3]	[134.7]	[32.7]

Note: Parentheses include the standard errors and brackets the t-values of the estimates.

TABLE 5. GARCH(1,1)-*t* Estimates

Estimates	DAX	Dow Jones	FTSE100	MIB30
α_0	1.29×10^{-4}	5.10×10^{-7}	3.61×10^{-7}	1.04×10^{-5}
	(5.34×10^{-5})	(2.04×10^{-7})	(1.90×10^{-7})	(2.94×10^{-6})
	[2.41]	[2.50]	[1.90]	[3.54]
α_1	0.062	0.046	0.042	0.098
	(0.010)	(0.008)	(0.008)	(0.028)
	[5.99]	[5.85]	[5.60]	[5.72]
β_1	0.915	0.929	0.948	0.833
	(0.013)	(0.011)	(0.009)	(0.028)
	[68.3]	[84.96]	[103.2]	[29.3]

Note: Parentheses include the standard errors and brackets the t-values of the estimates.

data sets analyzed show the stylized facts typical of financial returns series: heavy tails, persistence, heteroskedasticity, strong sample autocorrelations in the squared returns distribution, and asymmetry.

B. Estimation of GARCH Models

Now the results of the empirical applications of the models described in section I are shown to represent the qualitative features of the financial returns datasets. In particular, three different formulations of the GARCH family are fitted: the standard GARCH(1,1) with normal conditional distribution, the GARCH(1,1)-*t* with *t* student conditional distribution, also known as the fat-tailed GARCH, and the

TABLE 6. EGARCH(1,1) Estimates

Estimates	DAX	Dow Jones	FTSE100	MIB30
α_0	–0.309	–0.524	–0.247	–1.053
	(0.035)	(0.054)	(0.040)	(0.172)
	[–8.94]	[–9.66]	[–6.13]	[–6.11]
α_1	0.206	0.195	0.115	0.260
	(0.022)	(0.020)	(0.018)	(0.030)
	[9.44]	[9.62]	[6.22]	[8.63]
γ	–0.271	–0.499	–0.529	–0.172
	(0.052)	(0.088)	(0.112)	(0.053)
	[–5.19]	[–5.66]	[4.71]	[–3.28]
β_1	0.965	0.960	0.983	0.900
	(0.005)	(0.005)	(0.003)	(0.019)
	[179.1]	[195.3]	[292.5]	[48.1]

Note: Parentheses include the standard errors and brackets the t-values of the estimates.

EGARCH(1,1), with normal conditional distribution. The autoregressive and moving averages of GARCH components have been chosen by AIC, Akaike Information Criterion (Akaike [1974]).

Tables 4, 5 and 6 report the estimates and their standard deviations of GARCH(1,1), GARCH(1,1)-*t* and EGARCH(1,1) models. In general the parameter estimates are significantly different from zero except for the parameter α_0 of the FTSE100 return series in the GARCH-t estimation.

Considering GARCH(1,1) and GARCH(1,1)-t, the sum $\alpha_1 + \beta_1$ which represents volatility persistence, takes on high values (from 0.931 to 0.996).

Regarding EGARCH(1,1) estimation, the parameter of asymmetry, γ, is negative in all series. This means that the impact of negative returns on expected volatility is greater than that of positive ones. In particular this impact is more significant for the Dow Jones and FTSE100 series. In order to assess and compare the goodness of fit for each GARCH type formulation fitted to the data, standardized returns are determined by volatility estimates y_t/σ_t (it is the so called "devolatilization" procedure), and then standardized returns are tested to establish whether they are gaussian white noise or IID (see, Lundbergh and Terasvirta [2001] for a recent and thorough survey about GARCH diagnostic tests).

The Log-Likelihood values are displayed in table 7. Compared to

TABLE 7. Log-Likelihood Ratio Statistics

	GARCH(1,1)	GARCH(1,1)-*t*	EGARCH(1,1)
DAX	–32,619.9	–32,584.4	–32,609.6
Dow Jones	–27,157.7	–27,104.6	–27,143.3
FTSE100	–27,170.3	–27,161.3	–27,170.6
MIB30	–28,227.0	–28,212.1	–28,231.5

TABLE 8. Kurtosis Values for Adjusted Returns and Standardized Residuals

	y_t	GARCH(1,1)	GARCH(1,1)-*t*	EGARCH(1,1)
DAX	5.667	4.118	3.065	4.064
Dow Jones	8.058	4.707	3.018	4.466
FTSE100	4.389	3.514	3.011	3.651
MIB30	4.415	3.578	3.003	3.653

the standard GARCH(1,1) model, Log-likelihood values point out that fat-tailed GARCH(1,1)-*t* better estimate the returns of all four indexes. The EGARCH(1,1) model appears better than the standard version only for the DAX and Dow Jones return series, while for FTSE100 it shows exactly the same fitting, the worst performance being for the MIB30 returns. In particular, it is evident that as long as the asymmetric effect (see table 7) weakens, the EGARCH performs as well as the standard one, and sometimes worse. Finally, it should be noted that between GARCH(1,1)-*t* and EGARCH(1,1) the former always performs better. However, it is useful to remember that the formulation for the EGARCH model is different from that of GARCH and GARCH-*t*: the EGARCH models absolute shocks while the others model squared shocks. As a consequence comparison among the different likelihood function values is not very reliable. For this reason, in this paper work, alternative diagnostic tools such as the kurtosis values, the LB test for whiteness of residuals and the asymmetry test are utilized.

Table 8 shows kurtosis values for standardized filtered returns. GARCH(1,1)-*t* standardized returns have been "normalized" through the procedure suggested by Shephard (1996) and Kim et al. (1998) to compare models. Kurtosis estimates allow us to evaluate model adequacy to capture thick-tail unconditional distributions. For a perfect

TABLE 9. LB Statistics for Standardized Residuals and Their Squared Values

	GARCH(1,1)	GARCH(1,1)-*t*	EGARCH(1,1)
LB(30)			
DAX	43.589	43.392	44.338
Dow Jones	38.437	37.652	44.317
FTSE100	27.078	27.518	26.981
MIB30	35.522	35.440	35.132
$LB^2(30)$			
DAX	25.584	28.835	41.420
Dow Jones	15.038	20.814	22.033
FTSE100	20.637	20.403	22.563
MIB30	27.635	27.923	36.180

Note: LB is for statistics on standardized residuals and LB^2 is for their squared values. The critical value for the above LB statistics is $\chi^2(28)= 41.337$.

fit, kurtosis should be equal to three (the gaussian distribution kurtosis). As before, GARCH(1,1)-*t* guarantees the best fit, capturing the whole unconditional kurtosis. EGARCH, as already explained in section II.B, performs better than the standard model, whenever asymmetry is strong, i.e., for DAX and Dow Jones, while no remarkable improvement when asymmetry is low.

The Ljung Box test for autocorrelations in simple standardized returns (see table 9), shows the presence of a residual linear structure not captured by any model for DAX returns. Concerning the other datasets, standardized residuals are distributed as white noise, except for the EGARCH model applied to Dow Jones returns. As for the squared standardized residuals, the LB test does not indicate any significant autocorrelations, but for the EGARCH model applied to DAX returns. In short, the GARCH standard and the GARCH-*t* models guarantee a suitable goodness of fit, producing approximately IID standardized residuals, while the EGARCH model reveals some limits in fitting the data.

The Engle and Ng (1993) diagnostic tests are applied in order to check whether the EGARCH specification is adequate to describe returns asymmetry: the sign bias test (SBT) is a *t*-test which allows to verify the null hypothesis that $y_t^2/\hat{\sigma}_t^2$ are independent from $y_t/\hat{\sigma}_t$ signs; the negative size bias test (NSBT) and the positive size bias test (PSBT) are the *t*-tests of the null hypothesis that $y_t^2/\hat{\sigma}_t^2$ are

TABLE 10. EGARCH(1,1) Model Asymmetry Tests

	SBT	NSBT	PSBT
DAX	0.107	0.651	–1.548
Dow Jones	0.357	–0.429	–1.765
FTSE100	0.630	0.975	–0.379
MIB30	0.664	0.835	–0.112

Note: See Engle and Ng (1993) for the details regarding these tests.

independent from negative and positive $y_t/\hat{\sigma}_t$ shocks. As it is apparent in table 10, the EGARCH standardized residuals do not present any remaining asymmetry, showing that Nelson (1991) model is able to capture the leverage effect. Finally it can be deduced that the GARCH-*t* model fits returns better than the standard GARCH, describing jointly the tail thickness of the unconditional distributions and the squared returns autocorrelations, while the EGARCH describes asymmetry but not unconditional kurtosis completely. Thus a fat-tailed EGARCH model is strongly recommended for the analyzed datasets.

C. Estimation of SV models

In this subsection, the standard SV with normal conditional distribution (SV), the SV-*t* with *t* student conditional distribution (SV-*t*) and the asymmetric SV model (ASV) with normal conditional distribution are fitted. All models are estimated using the QML approach via the Kalman Filter suggested by Harvey et al. (1994) and Harvey and Shephard (1996).

Tables 11, 12 and 13 show the estimates and their standard deviations of the SV, SV-t and ASV models. It is important to note that estimates of the persistence parameter ϕ are very high in all series, even if significantly different from one (this means that volatility cannot be considered a random walk process). In particular, estimates of ϕ obtained from the application of SV and SV-t are very similar and are higher than the estimates obtained from ASV. This is mainly due to the greater number of parameters in the ASV model.

Referring to the asymmetric effect, the results obtained for the ASV model are very similar to those obtained for the EGARCH model. The

TABLE 11. SV Model Estimates

	ϕ	σ_η	α
DAX	0.989	0.115	–0.013
	(0.003)	(0.017)	(0.000)
Dow Jones	0.9942	0.0823	–0.0569
	(0.0025)	(0.0158)	(0.000)
FTSE100	0.9930	0.0818	–0.0661
	(0.0025)	(0.0131)	(0.000)
MIB30	0.971	0.153	–0.037
	(0.006)	(0.021)	(0.000)

Note: Parentheses include the standard errors of the estimates.

TABLE 12. SV-*t* Model Estimates

	ϕ	σ_η	α	ν	$k \sim t$
DAX	0.990	0.112	–0.093	37.58	3.881
	(0.003)	(0.017)	(0.033)		
Dow Jones	0.995	0.078	–0.051	15.43	4.887
	(0.002)	(0.016)	(0.026)		
FTSE100	0.993	0.076	–0.007	18.88	3.294
	(0.003)	(0.014)	(0.030)		
MIB30	0.973	0.146	–0.235	13.60	3.771
	(0.006)	(0.022)	(0.064)		

Note: Parentheses include the standard errors of the estimates.

parameter ρ, which represents the correlation between ε_t and η_t and hence the asymmetric effect, is negative in all series. Moreover, in its absolute value this coefficient is higher for Dow Jones and FTSE100. Table 13 reports kurtosis values for standardized returns after estimating the three models. SV-*t* standardized returns have been transformed as in the GARCH-*t* case. Consistent with the theoretical analysis outlined in section II, the standard model guarantees the best unconditional kurtosis fitting, so it is not necessary to resort to fat-tailed conditional distribution to replicate returns leptokurtic distribution. The SV asymmetric is the worst performer among the three models in describing outliers.

The Coefficient of Variation (CV) value, calculated as

TABLE 13. Kurtosis Values for Adjusted Returns, SV, SV-*t*, and ASV Models

	y_t	SV-N	SV-*t*	ASV
DAX	5.667	3.325	3.419	3.689
Dow Jones	8.058	2.872	3.840	4.449
FTSE100	4.389	3.078	3.048	3.363
MIB30	4.415	2.992	2.800	3.724

TABLE 14. Coefficient of Variations for SV, *SV-t* and ASV Models

	SV-N	SV-*t*	ASV-N
DAX	0.266	0.878	1.117
Dow Jones	1.013	0.840	0.925
FTSE100	0.164	0.513	0.646
MIB30	0.183	0.492	0.561

$\exp(\sigma_\eta^2/(1-\phi^2))$ is a measure of the relative strength of the level of conditional heteroskedasticity of returns series (Jacquier et al. [1994], Sandmann and Koopman [1998]). In particular, CV values around 1 indicate pronounced relative strength of the stochastic volatility process while values near to 0 signify that the model is close to constant volatility. Except in the case of the Dow Jones estimate, the standard SV coefficient of variation (see table 14) is too low to describe the heteroskedasticity shown by empirical returns. On the other hand, SV- and SV asymmetric models present CV values which are more consistent with the strong heteroskedasticity of the data.

The LB test for autocorrelations in standardized prewhitened returns displays significant values for DAX returns, regardless of the model applied. This result is probably due to the presence of a residual linear autocorrelation in the standardized series. As for other datasets, the LB test does not reject the white noise hypothesis for the standardized residuals. Besides, the LB test shows the standard SV model failure in representing the second moment structure of returns, while SV-*t* and SV asymmetric specifications do not reject the IID hypothesis. For the MIB30 dataset none of the models can describe the strong squared returns autocorrelation (see table 15).

Finally, in table 16 results of the Engle and Ng (1993) diagnostics

TABLE 15. LB Statistics for Standardized Residuals and Their Squared Values

	SV-N	SV-*t*	ASV-N
LB(30)			
DAX	44.218	43.360	44.464
Dow Jones	39.317	37.358	38.463
FTSE100	25.288	23.484	24.300
MIB30	33.059	25.724	25.668
$LB^2(30)$			
DAX	58.759	24.422	29.534
Dow Jones	24.050	35.924	19.595
FTSE100	26.590	28.444	30.368
MIB30	69.928	76.036	63.999

Note: LB is for statistics on standardized residuals and LB^2 is for their squared values. The critical value for the above LB statistics is $\chi^2(28)= 41.337$.

TABLE 16. ASV-N Model Asymmetry Tests

	SBT	NSBT	PSBT
DAX	–0.601	0.628	–2.731
Dow Jones	–1.172	–2.919	–2.694
FTSE100	–0.159	0.459	–1.161
MIB30	0.319	–1.932	1.897

Note: See Engle and Ng (1993) for the details regarding these tests.

tests are presented for the SV asymmetric model. Although first proposed and applied to GARCH asymmetric models, the three tests can also be applied to volatility models which are not members of the GARCH family (Engle and Ng [1993]). Results show a good performance for the asymmetric model in catching the FTSE100 and MIB30 returns asymmetry, and partial and total inadequacy in capturing the same, strongly present effect for DAX and Dow Jones returns. To summarize, diagnostic tests show that the standard version of the SV model is adequate to explain unconditional kurtosis of the datasets analyzed. For more leptocurtic returns distributions, as it is often encountered in literature (see, e.g., Taylor (1986), Bollerslev et al. (1994), Harvey et al. (1994), Shephard (1996), Drobetz and Zimmermann (2003), Engle and Patton (2001) and Peters (2001) among others), probably a SV-*t* or more fat-tailed model would be a better choice. On the other hand, the standard SV model cannot adequately

TABLE 17. Kurtosis Values for Standardized Residuals of All Models

Model	DAX	Dow Jones	FTSE100	MIB30
GARCH(1,1)	4.118	4.707	3.514	3.578
SV-N	3.325	2.872	3.078	2.992
GARCH(1,1)-*t*	3.065	3.018	3.011	3.003
SV-*t*	3.419	3.840	3.048	2.800
EGARCH(1,1)	4.064	4.466	3.651	3.653
ASV-N	3.689	4.449	3.363	3.724

describe conditional heteroskedasticy and squared returns autocorrelations, while the SV-*t*, although it performs slightly worse than the standard formulation in explaining the unconditional kurtosis, does not reveal any shortcoming in describing heteroskedasticity and second moment autocorrelations. The only exception is the MIB30 index which squared returns autocorrelations could not be described by any model. Finally, the asymmetric model has difficulty in capturing unconditional kurtosis, especially the asymmetric effect when it is strong (as in the DAX and MIB30 case).

D. Comparing performances

First of all, the models' adequacy to describe the typical thick-tails feature are compared. In particular, the kurtosis values of the standardized returns y_t/σ_t through the volatilities estimated by the application of each model are considered. The standardized returns should be approximately distributed as Gaussian white noises. The more the estimated kurtosis approaches the Gaussian kurtosis value of 3, the more the model can be considered adequate to capture the excessive number of outliers typical of unconditional distributions returns. Comparing the two standard formulations, the SV model shows values of kurtosis closer to the Gaussian kurtosis value of 3 than the GARCH one, indicating a better goodness of fit in representing fat tails (table 17).

These results confirm those obtained by other authors, such as Geweke (1995), Shephard (1996), Kim et al. (1998), who compared goodness of fit between GARCH and SV models using different methods. Asymmetric SV is slightly better than EGARCH in describing fat tails, even though it is not as good as the standard model. Finally, it is important to note that GARCH-*t* model is not only better able to

TABLE 18. Mean Absolute Deviations Between Induced ACFs and Squared Returns ACFs

Model	DAX	Dow Jones	FTSE100	MIB30
GARCH(1,1)	0.172	0.285	0.287	0.102
SV-N	0.036	0.058	0.080	0.044
GARCH(1,1)-*t*	0.154	0.142	0.217	0.148
SV-*t*	0.042	0.093	0.026	0.060
EGARCH(1,1)	0.079	0.099	0.089	0.070
ASV-N	0.011	0.005	0.018	0.008

capture empirical leptocurtosis (fat-tails) than the SV-*t* model, but is also the best in relation to the other models.

In order to compare the fit of the models in describing autocorrelations shown by squared returns, the autocorrelation function (ACF) and induced ACF are calculated by applying the theoretical equations provided in literature (Bollerslev [1986], Taylor [1994], He et al. [1999]), using parameter estimates and on the basis of a minimum distance criterion (mean absolute deviations), the best-fitting model is established.

Results reported in table 18 show that SV models dominate GARCH in describing squared empirical ACFs. Likewise, among the different specifications considered here, asymmetric models guarantee, for each class of models, the best goodness of fit regarding the specific returns feature analyzed. Besides, heavy tailed distributions perform better than standard formulations, and overall it is important to highlight that the best GARCH-type model (EGARCH) under-performs even the worst SV model (SV_{-t}). Furthermore, see figure 1 of the ACFs built from the parameters estimates of EGARCH (short dash line) and asymmetric SV (long dash line) compared with sample squared returns autocorrelations (dots) for the four indexes analyzed.

Tables 10 and 16 provide results of Engle and Ng (1993) tests applied to standardized residuals from EGARCH and SV asymmetric models. As shown by the reported values, EGARCH captures the leverage effect better than the SV asymmetric model. Indeed, while for FTSE100 and MIB30 indexes both models ensure the same fitting, for the DAX and Dow Jones indexes, i.e., the two more asymmetric datasets, tests show large values for the SV model, while for EGARCH no significant values are presented.

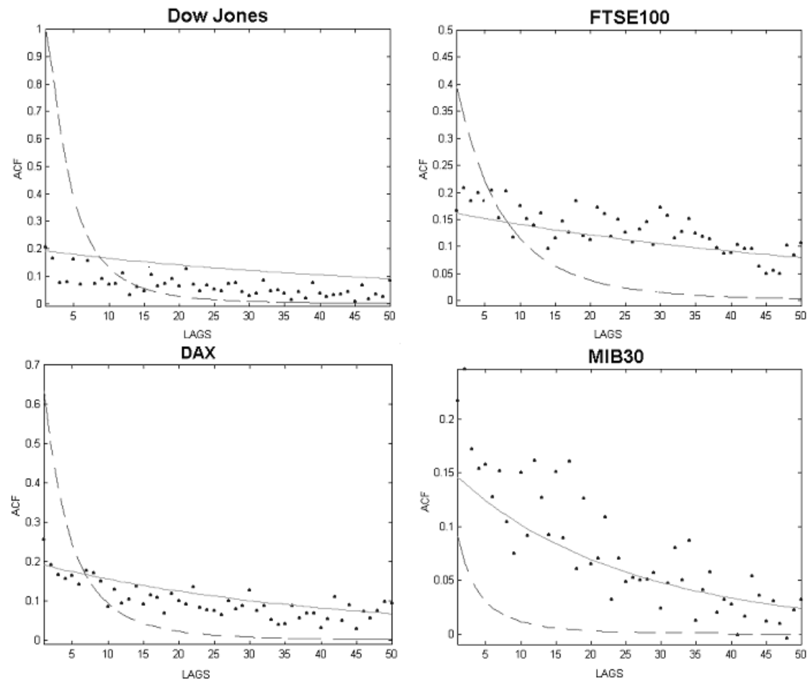

Figure 1.—ACFs Based on EGARCH (dashed line) and Asymmetric SV Model (line) vs. the Empirical Squared Autocorrelations (dots).

IV. Conclusion

On the basis of the results which emerge while using four different data sets of financial returns, it is possible to conclude that the Stochastic Volatility model, in its standard formulation, provides a better goodness of fit than the standard GARCH one. In particular, the normal SV dominates the normal GARCH both in describing heavy tailed unconditional returns and in representing the strong and slow decaying squared returns sample autocorrelations. As regards other more complex formulations, the outcomes are not so clearly in favour of SV models: GARCH-t formulation fits outliers better than SV-t, while it proves not to be as effective as SV in explaining squared autocorrelations.

Finally, asymmetric models, i.e., EGARCH and SV asymmetric, are not as good as the previous ones in describing fat tails, but are very suitable in approximating squared returns ACFs. Asymmetric SV is superior in capturing heavy tails and autocorrelations, while EGARCH is better in describing the asymmetric effect when this is particularly strong.

These results provide a preliminary analysis for the choice of the best model to forecast the volatility of financial returns. In fact, when the return distribution is characterized by fat-tails (or asymmetry) the GARCH-t (or EGARCH) models are recommended. Otherwise, when return distribution is normal, the SV model is more adequate.

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