Testing Asset Pricing Models: The Case of The Athens Stock Exchange*

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This article applies a conditionally heteroskedastic asset pricing model to describe the time variation in the first and second moments of asset returns in an interdependent way in the emerging capital market of Greece. Depending on the observability of the factors and under the chosen parameterization, it is possible to derive tests to address economically important questions that the models impose on the risk-return relationship. We apply the derived tests on the nine sectorial portfolios and the value-weighted index of the Athens Stock Exchange over the period 1985-1997. The evidence from the unconditional and conditional CAPM, with the Value-Weighted Index as a benchmark portfolio, suggests the inefficiency of the Index. On the other hand, the dynamic latent factor model, considered here, describes sectorial returns in a much better way. However, there is still a shadow of doubt on the hypothesis that the price of risk is common across assets. (JEL G12)

I. Introduction

Since its introduction in the early 1960s, the Sharpe (1964)-Lintner (1965) Capital Asset Pricing Model (CAPM) has been one of the most challenged topics in financial economics. In its simplest form, the CAPM predicts that the expected return on an asset above the risk-free

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rate is linearly related to the nondiversifiable risk, which is measured by the asset's beta with an intercept of zero. However, empirical evidence rejects the hypotheses of the zero intercept (direct test) (e.g., Gibbons, Ross, and Shanken [1989]) and-or that betas explain the cross-sectional variation in average returns (indirect test); (e.g., Fama and MacBeth [1973]). MacKinlay (1995) divided the explanations for violations of the model into two categories: non-risk-based alternatives and risk-based alternatives. The non-risk based category includes biases introduced by market frictions, the presence of irrational investors, data-snooping biases, biases in computing returns, transactions costs and liquidity effects, and market inefficiencies. The risk-based category includes multi-factor asset pricing models developed under investors' rationality and perfect capital markets. For this category, the source of deviations from the CAPM is either missing risk factors, as in Roll (1977), or the fact that the proxy of the market portfolio is inadequate, e.g. the proxy portfolio is not on the efficient frontier.

Recently, the emphasis has shifted towards inter-temporal asset pricing models in which agents' decisions are based on the distribution of returns conditional on the available information, which is obviously changing. This is partly motivated by the fact that financial markets volatility changes over time. However, it was not until Engle's (1982) work on Autoregressive Conditional Heteroskedasticity (ARCH) and Bollerslev's (1986) Generalized ARCH (GARCH) that researchers were able to take into account the time variation in first and second moments of returns. Noticeably, the new approach has had some empirical success. For example, Ng, Engle, and Rothschild (1992) found that, unlike a static setting, the basic restrictions of a CAPM type model were not rejected with U.S. data when they allowed the variance and covariance of the assets to vary over time.

The Ng, Engle, and Rothschild (1992) paper is based on what is termed the factor GARCH model of Engle (1987) (see also Kroner [1987], and Lin [1992]), where the time variation in the conditional variance can be summarized by a few linear combinations of the observed returns which are strong GARCH in the sense of Drost and Nijman (1993). Alternatively, to get the time variation in the two first moments of returns, the latent factor model of Diebold and Nerlove (1989) can be adopted as extended by King, Sentana, and Wadhwani

(1994). This model is a traditional statistical factor analysis model in which the variances of the common factors are parametrized as univariate ARCH-type models. Notice, however, that according to Sentana (1998), any factor GARCH model is nested within the class of conditionally heteroskedastic latent factor models.

The main contribution of this paper is the application of the three main tests, i.e. the Wald, Likelihood Ratio, and Lagrange Multiplier, to assess the validity of two asset pricing models, namely the CAPM and APT, in unconditional as well as conditional versions. To do this, the theoretical setup in King, Sentana, and Wadhwani (1994) is adopted, which results in a conditionally heteroskedastic latent factor model. Hence, the time variation in the first and second moments of asset returns are modeled in an interdependent way. Furthermore, the connection of this model to a conditional CAPM, where the market portfolio is approximated by an unobservable factor, is explicitly considered. In fact, it is shown that this version of CAPM is observationally equivalent to a one-latent-factor APT model.

Another innovation of this paper is the fact that the observability of the factors is central in the analysis. This is because it alters the likelihood function of the observed data set and consequently affects the tests. Hence, depending on this and under the specific chosen parameterization, tests are derived that are easily computable in order to address economically important questions, such as whether the model prices on average assets correctly, whether the price of risk is common across assets, and whether diversifiable and non-diversifiable risk is priced. In terms of the last question, the idiosyncratic conditional variances are modeled as ARCH-type processes. This is a non-trivial extension, because it allows us to distinguish between the hypotheses of the model pricing assets correctly, on average, and the pricing of idiosyncratic risk. Furthermore, to estimate the unobservable factors, we use the Kalman filter. Notice that, provided the returns' first and second moments are correctly specified, the Kalman filter-based limiting factor representing portfolios are best, in the mean square error sense, for any conditional distribution of returns (see Sentana [1994]).

In an attempt to extend the admittedly limited existing literature on emerging markets, the above tests are applied to fortnightly returns of the nine Athens Stock Exchange (ASE) sectorial indices and the Value

Weighted Index from January 1985 through June 1997. Equity returns in emerging capital markets have different characteristics than the ones in developed countries. According to Bekaert and Harvey (1997) emerging market returns are characterized by higher sample average returns, low correlations with developed market returns, more predictable returns, and higher volatility. The extent to which these higher average returns and volatility comply with asset pricing models is something which is important in at least two respects. First, in determining the cost of capital in local emerging markets and, second, in evaluating asset allocation in an international environment. Within this framework, three of the four aforementioned issues, namely higher average returns, higher predictability, and volatility, can be addressed. Furthermore, the multivariate setup we employ helps us to address questions such as the market integration, as well as the intertemporal variation, of the asset covariance, which are central in asset pricing models.[1]

The results indicate that the unconditional or conditional versions of the CAPM, with the VW Index as a benchmark portfolio, is rejected by the data. This is mainly due to the fact that the VW Index is a poor approximation to the market portfolio. However, it seems that the nine sectorial ASE excess returns comply with a conditionally heteroskedastic latent factor model. This model can also be interpreted as a CAPM model, with the market portfolio being approximated by an unobservable factor.

The organization of the article is as follows: In section II.A, the APT model theory is presented and the economic questions we would like to address are specified. The econometric methodology is presented in section II.B, whereas the tests for the hypotheses under consideration are discussed in sections II.C and II.D. Section III presents the empirical results. Section IV concludes.

1. Apart from the above-mentioned work, where ASE is examined as part of a multicountry study, very few empirical applications for ASE exist. These are mainly concentrated on the univariate characteristics of the Value Weighted Index; e.g., Alexakis and Petrakis (1991), Alexakis and Xanthankis (1995), and Koutmos, Negakis, and Theodossiou (1993).

II. Theory, Estimation, and Testing

A. Conditional APT Model

The theoretical asset pricing model is based on King, Sentana, and Wadhwani (1994), where more details can be found. The model is based on a world with an infinite number of primitive assets. The gross return of asset *i* in period *t*, $R_{i,t}$ (i=1,2,...) is generally uncertain since the asset is risky. However, the existence is assumed of a safe asset, whose return, $R_{0,t}$, is determined at the end of period t–1 and consequently is known to agents. Let $u_{i,t}$ be the unanticipated component of asset *i's* return as of time t–1,that is, $u_{i,t} = R_{i,t} - v_{i,t}$ where $v_{i,t} = E(R_{i,t} \mid I_{t-1})$ and I_{t-1} is the relevant information set which contains at least the past values of asset returns. The basic assumption made on the stochastic structure of asset returns is that $u_{i,t}$ has a conditional factor representation, so that it can be written:

$$u_{i,t} = \beta_{i,1,t} f_{1,t} + \beta_{i,2,t} f_{2,t} + ... + \beta_{i,k,t} f_{k,t} + \eta_{i,t} \ (i = 1,2,...)\,, \tag{1}$$

where $f_{j,t}$ (j = 1,2,...,k) are common factors which capture systematic risk affecting all assets, $\beta_{i,j,t} \in I_{t-1}$ (i = 1,2,...; j =1,2,...,k) are the associated factor loadings known at t–1, which measure the sensitivity of the asset to the common factor, while $\eta_{i,t}$ are idiosyncratic terms reflecting risk specific to asset *i*, which by definition are (conditionally) orthogonal to the common factors, i.e., $E(f_{j,t}\eta_{i,t}/I_{t-1}) = 0 \; \forall \; i, j$. Notice that the factor loadings are asset-specific. The unanticipated component, $u_{i,t}$, has by definition zero conditional mean. Consequently, $E(\eta_{j,t}/I_{t-1}) = 0 \; \forall \; i$, and $E(f_{j,t}/I_{t-1}) = 0 \; \forall \; j$. Without loss of generality, the factors are assumed to be conditionally orthogonal and to have time-varying variances, $\lambda_{j,t}$ (j =1,2,...,k). Idiosyncratic terms are assumed to be conditionally uncorrelated to each other, which corresponds to an exact conditional *k* factor structure (see Chamberlain and Rothschild [1983]), and have conditional variances $\gamma_{i,t}$. Notice that equation 1 is a statement about cross-sectional dependence of asset returns, and essentially says that the dimension of undiversifiable risk is *k* (see Chamberlain [1983]).

Within this framework it is possible to prove that (see King, Sentana, and Wadhwani [1994])

$$\mu_{i,t} = \nu_{i,t} - R_{0,t} = \beta_{i,1,t} f_{1,t} + \beta_{i,2,t} f_{2,t} + \ldots + \beta_{i,k,t} f_{k,t}$$
$$= \beta_{i,1,t}\pi_{1,t} + \ldots + \beta_{i,k,t}\pi_{k,t}\,. \tag{2}$$

Equation 2 is the main asset pricing equation which shows that risk premia can be written as an exact linear combination of the volatility of the common factors, with weights proportional to the corresponding factor loadings. This is very convenient for estimation purposes. The above equation can also be interpreted as saying that the risk premium of an asset is a linear combination of its factor loadings or betas, with weights common to all assets. These common weights $\pi_{j,t}$, can be understood as the risk premium associated with the j^{th} (limiting) factor representing portfolio, i.e., unit cost well-diversified portfolios of risky assets which have unit loading on only one factor and zero loadings on the other factors. Hence, asset risk premia are linear combinations of the k risk premia associated with the common factors. In fact, equation 2 can be understood as a conditional version of Ross (1976) exact APT. Given that $\lambda_{j,t}$ is the volatility of both factor j and its representing portfolio, and $\pi_{j,t}$ is the risk premium of this portfolio, $\tau_{j,t} = \pi_{j,t}/\lambda_{j,t}$ can be interpreted as the price of risk for that factor, i.e., the amount of expected return that agents would be willing to give away to reduce its variability by one unit.

Consequently, the model for the excess return, $r_{i,t} = R_{i,t} - R_{0,t}$, of the i^{th} asset is:

$$r_{i,t} = \beta_{i,1,t}\lambda_{1,t}\tau_{1,t} + \ldots + \beta_{i,k,t}\lambda_{k,t}\tau_{k,t} + \beta_{i,1,t} f_{1,t} + \ldots + \beta_{i,k,t} f_{k,t} + \eta_{i,t}$$
$$= \beta_{i,1,t}(\lambda_{1,t}\tau_{1,t} + f_{1,t}) + \ldots + \beta_{i,k,t}(\lambda_{k,t}\tau_{k,t} + f_{k,t}) + \eta_{i,t} \tag{3}$$
$$= \beta_{i,1,t}\left(R_{f_{1,t}} - R_{0,t}\right) + \ldots + \beta_{i,k,t}\left(R_{f_{k,t}} - R_{0,t}\right) + \eta_{i,t}\,.$$

where $R_{f_{1,t}}, \ldots, R_{f_{k,t}}$ are the returns of k limiting factor representing portfolio.

Now consider portfolios of primitive assets. Let $R_{p,t}$ be the gross return on a portfolio with weights $w_{p,t}$ (vector known at period t–1). The pricing model implies that the excess return of this portfolio, $r_{p,t}$, can be written as:

$$r_{p,t} = \beta_{p,1,t}(\lambda_{1,t}\tau_{1,t} + f_{1,t}) + \ldots + \beta_{p,k,t}(\lambda_{k,t}\tau_{k,t} + f_{k,t}) + \eta_{p,t},$$

where the factor loading coefficients, $\beta_{p,j,t}$, and the specific risk component, $\eta_{p,t}$, are linear combinations of the individual ' $\beta_{i,j,s}$ and $\eta_{i,t}$; however, the common factors, their variances, and prices of risk are the same as in equation 2. Therefore, in terms of factor structure, what is applicable to individual assets applies to portfolios as well.

It is worth emphasizing that, according to the model, risk prices depend on the factors, not on the assets, since otherwise there would be arbitrage opportunities. Furthermore, the also implies that specific risk, as measured by the volatility of the idiosyncratic terms, should not be priced, as it can be diversified away. Thus, its price should be zero. These fundamental restrictions shall be tested by employing the three main testing principles of Wald, Likelihood Ratio, and Lagrange Multiplier. The economic hypotheses of interest are the following.

Hypothesis 1: *Are risk prices different from zero*?

Hypothesis 2: *Does the model price, on average, asset efficiently*?

Hypothesis 3: *Is systematic risk valued in the same way across assets? In other words, is the market integrated?*

Hypothesis 4: *Is idiosyncratic risk priced*?

Before addressing the above questions, however, equation 2, which is a period-by-period cross-sectional restriction on the relative pricing of any subset of assets, should be transformed into an estimable model of the time-variation in risk premia. For this purpose, the evolution of $\beta_{i,j,t}, \tau_{j,t}, \lambda_{j,t}$, and $\gamma_{i,t}$ must be specified.

B. Econometric Methodology

To simplify notation, a one-factor pricing model is assumed for the rest of this paper. The factor loadings and the prices of risk are assumed time-invariant, i.e., $\beta_{i,t} = \beta_i$ and $\tau_t = \tau$, for all *t's*. Hence, for $k=1$ equation 3 becomes:

$$r_{i,t} = \beta_i \lambda_t \tau + \beta_i f_t + \eta_{i,t}, \text{ for every } i.$$

The above assumption is not as restrictive as it seems. To realize its implication, notice that the above equation can be written as:

$$r_{i,t} = \beta_{i,t} \tau_t + \beta_{i,t} f_t^* + \eta_{I,t},$$

where $\beta_{i,t} = \beta_i \lambda_t^{1/2}, \tau_t = \tau \lambda_t^{1/2}$ and $f_t^* = f_t / \lambda_t^{1/2}$. In other words, the assumption is observationally equivalent to a model in which the conditional variance of the factor is constant, but the betas of different assets on the factor change proportionately over time, and there is time-variation in the prices of risk (see King, Sentana, and Wadhwani (1994), and Demos, Sentana, and Shah (1994)).

Next, the temporal variation in the volatility of common and idosyncratic factors need to be specified. It is assumed that factor, λ_t, and idiosyncratic, $\gamma_{i,t}$, conditional variances are time-variant. Specifically, a *strong* ARCH-type parameterization, in the sense of Drost and Nijman (1993), is assumed. In fact, conditional variances are, in general, GQARCH(1,1) (quadratic GARCH) models of Sentana (1995). A comparative advantage of the GQARCH model is that it captures not only the autocorrelation in the stock market volatility, but also allows for asymmetric effects in the volatility response to positive and negative shocks of the same size. The asymmetric effect has been successfully documented in the U.S. (see Campbell and Hentschel [1992]), U.K. data (see Demos, Sentana and Shah [1994]) and the ASE (see Koutmos, Negakis, and Theodossiou [1993]).

The asymmetry effect can also be incorporated within other asymmetric GARCH models such as the Exponential GARCH of Nelson (1991) or the asymmetric power GARCH of Ding, Granger, and Engle (1993). However, these models impose an ARMA structure on a non-linear transformation of the conditional variance. This would make the evaluation of the unobservable factor-conditional variance via the Kalman filter extremely difficult. Consequently, the choice of GQARCH model not only incorporates the excess stock returns stylized facts but avoids, at a minimal cost, the difficulty with the Kalman filter estimates (see section II.D below).

C. Testing with Observed Limiting Factor Representing Portfolio

Assume for the moment that the limiting factor representing portfolio, $R_{f,t}$,is observed. If the excess return of the diversified-basis portfolio is added to the list of N portfolios under consideration, the excess returns in a vector notation are:

$$r_{f,t} = \beta(\lambda_t \tau + f_t) + \eta_t = \beta r_{f,t} + \eta_t ,$$

$$r_{f,t} = \lambda_t \tau + f_t , \tag{4}$$

$$\lambda_t = \theta_0 + \theta_1 f_{t-1} + \theta_2 f_{t-1}^2 + \varphi_1 \lambda_{t-1},$$

where r_t is a N x 1 vector of excess returns, β is the N x 1 vector of factor loadings, τ is the price of risk, η_t is the N x 1 vector of idiosyncratic errors, $r_{f,t}$ is the excess return of the limiting factor representing portfolio, and λ_t is the GQARCH(1,1) conditional variance of the factor (see Demos and Sentana [1998]).

The joint likelihood function of r_t, $r_{f,t}$, conditional on I_{t-1}, can be factorized into the conditional of r_t given $r_{f,t}$ (and I_{t-1}) times marginal of $r_{f,t}$ (given I_{t-1}) i.e.

$$g(r_t, r_{f,t} \mid I_{t-1}) = g(r_t \mid r_{f,t}, I_{t-1}) g(r_{f,t} \mid I_{t-1}) .$$

Conditional on the past, the marginal of $r_{f,t}$ has mean $\tau\lambda_t$ and variance λ_t, whereas the conditional of r_t has mean vector $\beta r_{f,t}$ and diagonal covariance matrix Γ_t. Thus, assuming conditional normality, the log-likelihood function for $r_t, r_{f,t}$ can be written as:

$$L = -\frac{T(N+1)\ln 2\pi}{2} - \frac{1}{2}\sum_{t=1}^{T}\left\{\sum_{i=1}^{N}\left[\ln\gamma_{i,t} + \frac{\left(r_{i,t} - \beta_i r_{f,t}\right)^2}{\gamma_{i,t}}\right]\right\}$$

$$-\frac{1}{2}\sum_{i=1}^{T}\left[\ln\lambda_t + \frac{\left(r_{f,t} - \tau\lambda_t\right)^2}{\lambda_t}\right].$$

Such a factorization performs a sequential cut on the joint log-likelihood function which makes $r_{f,t}$ strongly exogenous for the parameters in β and Γ_t (see Engle, Hendry and Richard[1983], and Demos and Sentana [1998]). As a result, if it is further assumed that Γ_t is time-invariant, i.e., $\Gamma_t = \Gamma$ for all *t's*, the parameters in vector β can be estimated by univariate regressions of $r_{i,t}$ on $r_{f,t}$. Such an estimator is consistent and furthermore, heteroskedasticity robust standard errors are not needed as the regressors are strictly exogenous. However, in case that $\gamma_{i,t}$ follow a time varying process, e.g., ARCH type process, the OLS $\beta_i s$' estimators are inefficient.

If the asset pricing restrictions in this model do not hold, due to the fact that average expected returns on the assets are unrestricted, equation (4) becomes:

$$r_t = a + \beta(\lambda_t\tau + f_t) + \eta_t = a^* + \beta r_{f,t} + \eta_t$$

$$r_{f,t} = a_f + \lambda_t\tau + f_t,$$

where α is a vector containing the Jensen's alpha for each asset, and $\alpha^* = \alpha - \beta\alpha_f$.

The joint likelihood function can again be factorized into the marginal component for $r_{f,t}$, and the conditional component for r_t, given $r_{f,t}$. Thus α_f, τ and the parameters in λ_t can again be effectively estimated by a univariate GARCH in Mean type model with a constant in the mean, whereas the *ML* estimates of α^*, β and Γ can be obtained by N univariate OLS regressions of $r_{i,t}$ on a constant and $r_{f,t}$. The *LR* test, in this case, for the hypothesis of unbiased pricing, hypothesis H_2 in section II.A, takes the $LR = T\sum_{i=1}^{N}(\ln\hat{\gamma}_{\mathrm{i}} - \ln\tilde{\gamma}_i)$,where $\hat{\gamma}_{\mathrm{i}}$, and $\tilde{\gamma}_{\mathrm{i}}$ are the restricted and unrestricted estimated idiosyncratic variances for asset i, which under the null follows a χ^2 distribution with N degrees of freedom; e.g. Engle (1984) and Sentana (1997). Notice that if a non-diagonal idiosyncratic covariance matrix is allowed, i.e., $\gamma_{i,j} \neq 0$, the analysis above goes through unchanged. The *LR* test now has the form $T\left(\ln\left|\hat{\Gamma}\right| - \ln\left|\tilde{\Gamma}\right|\right)$, which is the *LR* version of the Gibbons, Ross, and Shanken(1989) Wald test for mean variance efficiency of a given portfolio.

In this setup it can be tested whether the market is integrated (*hypothesis* 3), i.e., if the price of risk is common across assets. Assume for the moment that it is not integrated, i.e., equation 4 becomes:

$$r_{i,t} = \beta_i\lambda_t\tau_i + \beta_i f_t + \eta_{i,t} = \beta_i r_{f,t} + \beta_i\tau_i^*\lambda_t + \eta_{i,t},$$

$$r_{f,t} = \lambda_t\tau + f_t$$

where $\tau_i^* = \tau_i - \tau$. Notice that the price of risk in this case is asset-dependent.

In this case $r_{f,t}$ is not even weakly exogenous for the parameters in β and Γ. Consequently, the application of the Wald or *LR* test will require the joint estimation of the whole system. However, under the null hypothesis of common price of risk, i.e., $\tau_i^* = 0$ for all *i's*, strong exogeneity is maintained. This makes the application of the *LM* procedure very attractive.

Hence, under the alternative hypothesis of $\tau_i^* \neq 0$, the conditional variance of the factor will have additional explanatory power, apart from the one through the factor-representing portfolio. The *LM* version of this test is distributed, under the null of common price of risk, as χ_1^2 (see Engle (1984)) and is *T*, the number of observations, times the R^2 from the regression of the residuals of equation (4) above on $r_{f,t}$ and λ_t. Notice that, under the assumption of diagonal idiosyncratic covariance matrix, Γ, the joint *LM* test is simply the sum of individual ones.

If the assumption of time-variation of the idiosyncratic conditional variances is also brought back, the strong exogeneity of the observed factor representing portfolio is maintained. Consequently, the maximum likelihood estimates for the parameters in β and Γ_t are obtained by *N* univariate maximum likelihood estimations of the asset excess returns, with the observed limiting factor portfolio excess returns as an explanatory variable, and ARCH-type errors. Whereas τ and the parameters in λ_t are estimated from an ARCH-M type maximization. Notice that there is no gain in system estimation due to the diagonality of Γ_t.

In this setup it can be tested if the model prices, on average, assets correctly by testing the significance of a constant in each of the *N* ARCH-type regressions, H_2 in section II.A. This can be done by employing the *LR* test. Again, due to the assumed diagonality of the variance covariance idiosyncratic matrix, the joint *LR* test is the sum of the individual ones. Furthermore, the *LM* test for the integration of the market, Hypothesis H_3 in section II.A, is now TR^2 from the regression of the standardized residuals on $\left(\gamma_{i,t}\right)^{-.5} r_{f,t}$ and $\left(\gamma_{i,t}\right)^{.5} \lambda_t$ .Under the null, of common price of risk, the *LM* test is distributed asymptotically as χ_1^2.

Having time-variant idiosyncratic variances helps to test the model in another direction as well. According to the pricing model, idiosyncratic risk should not be priced, as individuals can diversify it away (Hypothesis H_4 in section II.A). Having a constant idiosyncratic variance is equivalent to the assumption that the model is pricing assets correctly, on average. Hence, allowing time-variant idiosyncratic variances helps us test if a significant constant in the time invariant specification is due to the model-pricing assets inefficiently or due to idiosyncratic variances being priced. The *LM* test for this hypothesis is TR^2 from the regression of the standardized residuals on $\left(\gamma_{i,t}\right)^{-.5} r_{f,t}$ and

$\left(\gamma_{i,t}\right)^{.5}$, which, under the null of zero price of risk, is distributed asymptotically as χ_1^2.

D. Testing with Unobserved Limiting Factor Representing Portfolios

The equation for the *N* available assets can be written as:

$$r_t = \beta \lambda_t \tau + \beta f_t + \eta_t \, . \tag{5}$$

Models such as that in equation 5 are estimated for all *N* assets simultaneously by maximum likelihood under the assumption of joint conditional normality for r_t. Ignoring initial conditions, the log-likelihood function takes the form:

$$c - \frac{1}{2}\sum_{t=1}^{T}\left\{\ln\left|\lambda_t\beta\beta' + \Gamma_t\right| + tr\left[(\lambda_t\beta\beta' + \Gamma_t)^{-1}(r_t - \beta\lambda_t\tau)(r_t - \beta\lambda_t\tau)'\right]\right\},$$

where $c = -TN\ln 2\pi/2$. Since the first-order conditions are particularly complicated in this case (see Demos and Vasillelis [1997]), a numerical approach is usually required.

At this point it is important to emphasize that the factor is an unobservable random variable. This is because in empirical applications there is data on a finite number of assets and, consequently, from the econometrician point of view f_t is random, whose returns must be proxied by those of representing portfolios obtained from the collection of assets under consideration. As now f_t can take different values in different realizations, it seems natural to use a signal extraction approach. In Sentana (1994), it is proved that the basis portfolios generated by the Kalman filter yield the best, in the mean square error sense, factor-representing portfolios possible for any conditional distribution of returns. Intuitively this result is based on the following arguments.

Any conceivable factor-representing portfolio will be a linear combination of the assets under consideration with, possibly, time-varying weights. It is well known that the Kalman filter estimates are best, in an unconditional sense, within the class of linear predictors (e.g., Harvey [1989]). This can be generalized in a conditional setup; see theorem 1 in Sentana (1994). Hence, the optimality of the Kalman filter estimates stems from the fact that they can be interpreted as the conditionally linear least-squares projection on the conditionally linear space generated by the unexpected part of asset returns.

Consequently, taking f_t as the state variable, equation 5 can be understood as the measurement equation with $f_t = f_t$ as the transition equation, which does not contain any mean dynamics; see Diebold and Nerlove (1989), and Harvey, Ruiz, and Sentana (1992).

The prediction equations are (e.g., Harvey, Ruiz, and Sentana [1992], and Demos and Sentana [1998]):

$$f_{t/t} = \lambda_t \beta'(\lambda_t \beta\beta' + \Gamma_t)^{-1}(r_t - \beta\lambda_t \tau)$$

and

$$\lambda_{t/t} = \lambda_t - \lambda_t \beta'(\lambda_t \beta\beta' + \Gamma_t)^{-1}\beta\lambda_t .$$

Notice that since the transition equation is degenerate, smoothing of the factor is not necessary as its smoothed estimates will coincide with the updated ones, i.e., $f_{t|t} = f_{t|T}$ (see Diebold and Nerlove [1989]). Furthermore, notice that the weights used to construct the updated estimates are time-varying.

In order to avoid the non-measurability of the conditional variance with respect to the econometrician's information set, which is smaller than that of the agents, the correction of Harvey, Ruiz, and Sentana (1992) is adopted. For the GQARCH(1,1) case in particular, this can be achieved by an equation of the form (see Sentana [1995]):

$$\lambda_t = a_0 + a_1 E(f_{t-1}|r_{t-1}) + a_2\left[E\left(f_{j,t-1}|r_{t-1}\right)^2 + V\left(f_{j,t-1}|r_{t-1}\right)\right] + a_3\lambda_{t-1}, \quad (6)$$

i.e., the measurability of $\lambda_{j,t}$ with respect to r_{t-1} is achieved by replacing the unobserved factor by its Kalman filter estimate, and including a correction in the standard ARCH term which reflects the uncertainty of the factor estimate. In other words, instead of the standard ARCH term, $E\left(f_{j,t-1}^{2}|r_{t-1}\right)$ there is now the term $E^{2}(f_{j,t-1}|\, r_{t-1})+V(f_{j,t-1}\,|\, r_{t-1})$, where the correction term, $V\left(f_{j,t-1}|r_{t-1}\right)$, reflects the uncertainty about the factor estimate. Furthermore, an arbitrary element of the model is the scaling of the factor. This is resolved by assuming that the factor has a unit-unconditional variance. Notice that in such a case the α_0 is a free parameter, as it is given by one minus the sum of α_2 and α_3; see Sentana and Fiorentini (1997).

In terms of testing, let us first consider the application of the *LM* test to test any of the four hypotheses of section II.A. It must be noted that there are three complications associated with this task. First, the information matrix is not diagonal between the mean and variance parameters, unless $\tau = 0$. Second, there are not closed-form solutions for the first-order conditions for the maximum likelihood. The above-mentioned facts are particular to all conditionally heteroskedastic in mean models. Third, a fact which is associated to the model only, is that the factors are not observed. All these considerations make the application of the *LM* test rather cumbersome.

On the other hand, the application of the Wald test involves the inversion of the information matrix that is not block diagonal, estimated under the alternative. This is not an easy task, especially if the number of assets is large. Given that the model must be estimated under the alternative as well, it seems that the application of the *LR* test is easier compared to the other two. Under standard regularity conditions, its asymptotic distribution is the usual χ^2 with degrees of freedom in the number of restrictions.[2] In the empirical section, this last principle is employed to test the four hypotheses of interest.

III. Empirical Applications

For the empirical applications, data from the 9 sectors and the Value Weighted Index of the Athens Stock Exchange for the period January 1, 1985, until June 30, 1997, i.e., a total of 300 fortnightly returns, is

2. For the asymptotic distribution of all three tests, see Eagle (1984).

TABLE 1. The Athens Stock Exchange Since 1985

Year	Share Price Index	Transactions*	Market Value**	No.of Companies
1985	70.95	2.38	113.06	114
1986	103.86	4.53	156.61	114
1987	272.47	59.62	565.58	116
1988	279.65	44.38	598.43	119
1989	459.43	89.06	996.62	119
1990	932.00	608.69	2426.59	145
1991	809.71	437.55	2355.17	159
1992	672.31	307.29	2044.36	164
1993	958.66	637.08	3117.07	150
1994	868.91	1261.41	3577.77	196
1995	914.15	1408.47	4025.90	215
1996	933.48	1990.01	5944.78	235
1997	1479.63	5540.40	9600.00	245

Note: The Athens Stock Exchange since 1985. Source: Annual Economic Review National Bank of Greece (1998). A (*)in bn GRD and (**)base year 1990.

employed. For all portfolios, excess returns are considered over and above the safe rate of return, which in this case is the return on the 3-month Treasury Bills.

Security markets in Greece have been thin and limited primarily to a small market in equities. Despite recent growth, the Athens Stock Exchange has typically played only a minor role in Greek corporate finance, and only a small number of banks and insurance firms have consistently accounted for the bulk of its capitalization.[3] For most of the sample period, the number of industrial and commercial companies listed on the ASE was exceedingly small. As is apparent from table 1, overall, the number of listed companies, although doubled, remained for the major part of the sample period under 200. The importance of the ASE has grown dramatically since 1989, following a number of important measures such as the permission to set up brokerage firms, the establishment of a computerized central securities depository, the

3. According to OECD estimates, the ratio of market capitalization to GDP (15% for 1995) is one of the lowest in the OECD area (France 38%, Germany 26%, Portugal 23%, Spain 26%, Australia 115%). OECD Economic Survey. Greece (March 1995).

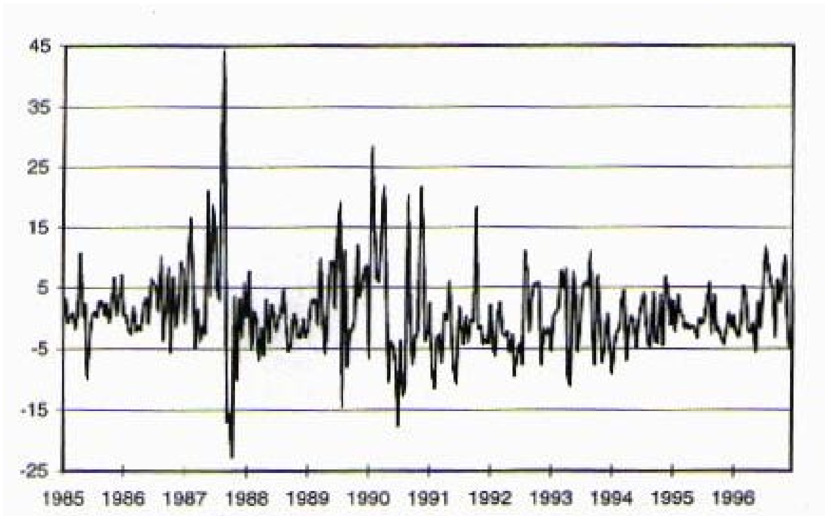

FIGURE 1.—Excess returns on the VW index.

introduction of an electronic trading system, the adoption of most of the EEC directives regarding stock exchange legislation, and the upgrading of the supervisory authority of the ASE, the Capital Market Committee, to an independent regulatory body. The development of the ASE until 1989 and its impact on the efficiency of the market could constitute an important limitation to this study, which could be overcome by new updated studies in years to come.

The excess returns of the VW Index are presented in figure 1. On a purely descriptive level, the second half of October 87 stands out with a return of –16%, although the minimum return over the whole sample, a decrease of 22.68%, occurred during the second half of November 87. However, preceding this period, the index had experienced a strong upward trend, reaching a maximum gain of 44.16% during the second half of September 87. Over the whole sample period, the mean excess return stands at .85% and the average standard deviation is 7.27%. Notice that during the decade 1985-95, Greek interest rates were well above 15%. Furthermore, there are periods of high volatility followed by quiet ones, which confirm the importance of modeling its time variation. Let us turn to a more detailed analysis.

A. Observed Factor Representing Portfolio

TABLE 2. Market Betas (Value Weighted Index)

OLS Regressions $r_{i,t}=\beta_i r_{VW,t}+\eta_{i,t}$ (i=1,...,9)			
Sector	β_i	t-statistic	R^2
Banks	.979	24.26	67.4
Investment	.878	17.75	50.8
Insurance	.977	15.34	43.4
Miscellaneous	.442	13.45	36.9
Holding	.860	13.56	37.5
Textile	.963	13.70	37.9
Construction	.724	14.53	40.6
Food	.719	15.58	44.1
Chemicals	.989	18.10	51.6

Note: $r_{i,t}$ is the excess return of the i^{th} sector, η_{it} is the idiosyncratic error. $r_{VW,t}$ is the excess return of the Value Weighted Index.

To start the analysis, the VW Index is assumed to be well-diversified. In terms of section II.C it is assumed that $r_{f,t} = r_{VW,t}$.

Unconditional CAPM

From section II.C above, it is seen that the factor sensitivities in a single observed factor model, imposing asset pricing restrictions, is obtained from univariate OLS regressions of the portfolio excess returns on the excess returns of the observed factor-representing portfolio. OLS estimators are efficient under the assumption of homoskedastic idiosyncratic errors and remain consistent under heteroskedastic ones. In this respect, the model is estimated as the traditional CAPM of Sharpe (1964) and Lintner (1965). The results are presented in table 2.

A simple, yet powerful, way to test the model is to include a constant in the univariate OLS regressions. Under the joint hypothesis that the VW Index is an efficient frontier portfolio and the CAPM holds, the nine intercepts should be statistically not different from zero; e.g., Gibbons, Ross, and Shanken (1989). Notice that these regression intercepts correspond to Jensen's alpha as defined in the portfolio performance literature, and measure average "abnormal" returns from the point of view of the model.The results of the estimated alphas and

TABLE 3. Jensen's α (Value Weighted Index)

OLS Regressions $r_{i,t} = \alpha_i + \beta_i r_{VW,t} + \eta_{i,t}$ $(i = 1,\dots,9)$		
Sector	α_I	t-statistic
Bank	1.188	4.20
Investment	.257	.71
Insurance	.578	1.34
Miscellaneous	.334	1.38
Holding	.387	.83
Textile	.630	1.36
Construction	1.026	2.83
Food	.922	2.74
Chemicals	.777	1.94

Note: $r_{i,t}$ is the excess return of the i^{th} sector, η_{it} is the idiosyncratic error. $r_{VW,t}$ is the excess return of the Value Weighted Index.

their standard errors are presented in table 3. In only 3 out of 9 cases is the alpha significantly different from zero. However, the joint tests are more conclusive. Under the assumption of diagonal idiosyncratic covariance matrix, the *LR* test yields 42.9, far away from the critical value of χ^2_9 at 1% which is 21.7. Also, the equivalent Wald test, which in this case is simply the sum of the individual squared *t*-statistics, is 43.39, highly significant. Furthermore, if a non-diagonal idiosyncratic covariance matrix is allowed, then the *LR* and the Wald tests take the values 34.29 and 36.33, respectively, which again are highly significant.

Conditional CAPM

Assuming the VW Index follows a GQARCH(1,1)-M process, but maintaining the hypothesis of time-constancy in the idiosyncratic variance, model (4) above can be efficiently estimated by 9 OLS regressions of the portfolios' excess returns on the excess returns of the VW Index, as in the previous section, and one GQARCH in Mean univariate maximum-likelihood estimation for the excess return of the VW Index. The results of the ML estimation for the VW Index are presented in table 4. Notice that persistence is quite high, .959, something which is well-documented internationally. Furthermore, the

TABLE 4. GQARCH(1,1)-M for the VW Index

$$r_{VW,t} = \tau\lambda_t + f_t,\ \lambda_t = \theta_0 + \theta_1 f_{t-1} + \theta_2 f_{t-1}^2 + \theta_3 \lambda_{t-1}$$

Parameter	Value	*t*-statistic
τ	.007	1.25
θ_0	1.340	1.90
θ_1	.532	2.13
θ_2	.151	3.54
θ_3	.808	19.96

Note: r_{VW},t is the excess return of the Value Weighted Index, f_t is the observable factor and λ_t is its conditional variance.

asymmetry parameter, θ_1, is significantly different from zero, but is positive, i.e., positive shocks have a higher impact on volatility than negative ones of the same size. This is the opposite from the asymmetric response in the U.S. and U.K. data (see Campbell and Hentschel [1992], and Demos, Shah, and Sentana [1994]), but in agreement with the Spanish and the Greek data (see Sentana [1997], and Koutmos, Negakis, and Theodossiou [1993]). However, the price of risk, τ, although positive, is statistically insignificant (*t-statistic* = 1.25).

In this setup, an interesting exercise can be performed, namely to test for integration in the ASE (see section II.C). For this case, the *LM* test is employed. The reason is that under the alternative, i.e., that the price of risk is different for the nine portfolios, $r_{f,t}$ is no longer weakly exogenous for the parameters β and Γ. Consequently, in order to form under this assumption the *LR* and Wald tests, one needs to estimate the full model by maximum likelihood. Hence, the use of the *LM* test is the appropriate one in this case. The test is the usual TR^2 from the regression of the residuals on the $r_{f,t}$ and its conditional variance. As Γ is diagonal, the joint *LM* test is the sum of the individual ones and is equal to 69.22, which is strongly significant, even at 1%.

Relaxing the assumption of constant idiosyncratic variances, but maintaining the zero correlation between idiosyncratic errors, it can be seen from section II.C that the *ML* estimates for *β's* can be obtained from 9 univariate *ML* estimations of the sectorial returns on the VW Index ones with GQARCH(1,1) errors. The estimated *β's* are presented

TABLE 5. Sectorial Portfolios Market betas (Value Weighted Index)

$$r_{i,t} = \beta_i r_{VW,t} + \eta_{i,t}, \; \gamma_{i,t} = \delta_{i,0} + \delta_{i,1}\eta_{i,t-1} + \delta_{i,2}\eta^2_{i,t-1} + \phi_{i,1}\gamma_{i,t-1}$$

$$r_{VW,t} = \tau\lambda_t + f_t, \; \lambda_t = \theta_0 + \theta_1 f_{t-1} + \theta_2 f^2_{t-1} + \theta_3\lambda_{t-1}$$

Sample Period 1985:01-1997:06

Sector	β_i	*t*-statistic
Banks	1.058	34.91
Investment	.828	16.44
Insurance	.964	20.88
Miscellaneous	.361	9.84
Holding	.710	9.44
Textile	.646	9.18
Construction	.742	17.57
Food	.781	14.54
Chemicals	.934	14.39

Note: $r_{i,t}$ is the excess return of the ith sector, $\eta_{i,t}$ is the idiosyncratic error, and $\gamma_{i,t}$ its conditional variance. $r_{VW,t}$ is the excess return of the Value Weighted Index, f_t is the observable factor, and λ_t is its conditional variance.

in table 5. Based on the discussion in section II.C, the estimated *β's* are not, in general, very different from the homoskedastic regression ones, with a correlation of .811. In fact, in only 3 cases, Banks, Miscellaneous and Textile, the *β's* from the static CAPM are more than 2 standard deviations from the dynamic ones. However, as expected, standard errors are considerably different.

The persistence in the conditional idiosyncratic variances range from .789 in Chemicals to .986 in Investment. Out of the 9 asymmetry parameters, only 4 are significantly different from zero, those in Banks and Food, with estimated values being negative, –.077 (*LR* = 7.66) and –.229 (*LR* = 9.89) respectively, and in Insurance and Miscellaneous with positive estimated values .33 (*LR* = 12.26) and 1.386 (*LR* = 5.22), respectively. Notice that asymmetries in the idiosyncratic conditional variances are on top of the asymmetry present in the market proxy.

Table 6 presents the tests associated with the four hypotheses of interest. As has been already discussed, the common price of risk is insignificantly priced. The *LR* test for this hypothesis is 1.57, highly

TABLE 6. Hypothesis Testing for Sectorial Portfolios (VW Index Model)

$$r_{i,t} = \beta_i r_{VW,t} + \eta_{i,t}, \gamma_{i,t} = \delta_{i,0} + \delta_{i,1}\eta_{i,t-1} + \delta_{i,2}\eta^2_{i,t-1} + \phi_{i,1}\gamma_{i,t-1}$$

$$r_{VW,t} = \tau\lambda_t + f_t, \lambda_t = \theta_0 + \theta_1 f_{t-1} + \theta_2 f^2_{t-1} + \theta_3\lambda_{t-1}$$

Hypothesis	Test Value	Critical Value Under the null	
Is Risk Priced	1.57	$\chi^2_{1.5\%} = 3.84$	
Non-Zero Constants in Mean	29.37	$\chi^2_{9.1\%} = 14.7$	$\chi^2_{9.5\%} = 16.9$
Is the Price of Risk Common	49.16	$\chi^2_{8.1\%} = 13.4$	$\chi^2_{8.5\%} = 15.5$
Is Idiosyncratic Variances Priced	20.29	$\chi^2_{9.1\%} = 14.7$	$\chi^2_{9.5\%} = 16.9$

Note: $r_{i,t}$ is the excess return of the ith sector, $\eta_{i,t}$ is the idiosyncratic error, and $\gamma_{i,t}$ its conditional variance (GQARCH). $r_{VW,t}$ is the excess return of the Value Weighted Index, f_t is the observable factor, and λ_t is its conditional variance (GQARCH).

insignificant. The joint *LR* test for the average efficient pricing of assets is 29.37, which is significant even at 1%. This time the *LM* test for the integration of the market is TR^2 from the regression of the standardized residuals on $(\gamma_{i,t})^{-.5} r_{f,t}$ and $(\gamma_{i,t})^{.5}\lambda_t$ (see section II.C above). The joint test, i.e. the sum of individual ones due to assumed diagonality of the idiosyncratic variance covariance matrix, is 49.16, significant at 1% ($x^2_{8.1\%}$ = 20,1). Again from section II.C, the *LM* test for the pricing of idiosyncratic risk is TR^2 from the regression of the standardized residuals on $(\gamma_{i,t})^{-.5} r_{f,t}$ and $(\gamma_{i,t})^{.5}$. The joint *LM* test is equal to 20.29 significant at 5% but not at 1%.

To conclude the above two subsections, it can be said that the CAPM, with the VW Index as a benchmark portfolio, fails to price assets correctly, on average, in either conditional or unconditional setup. This, of course, could mean that either the VW Index fails to be on the efficient frontier, perhaps because the weights of a small number of companies are “too” high (see Adcock and Clark [1999]), and/or there are missing risk factors. As a developing market is being considered, both explanations are quite plausible. Furthermore, in the conditional setup, the systematic risk does not have a common price across assets, and idiosyncratic risk is priced.

TABLE 7. Factor Loadings (Static Latent Factor)

$$r_{i,t} = \beta_i r_{f,t}, \ \eta_{i,t} \sim N(0, \gamma_i), \ (i = 1, \ldots, 9)$$

$$r_{f,t} = \tau + f_t, \ f_t \sim N(0,1)$$

Sector	β_i	*t*-statistic	$\tau\beta_i$	*t*-statistic
Banks	6.248	14.24	1.331	3.55
Investment	6.700	15.61	1.427	3.61
Insurance	7.477	14.02	1.592	3.58
Miscellaneous	3.791	14.58	0.807	3.58
Holding	6.509	12.23	1.386	3.54
Textile	8.394	13.84	1.788	3.57
Construction	5.631	13.21	1.199	3.52
Food	5.324	13.25	1.134	3.54
Chemicals	8.008	15.97	1.706	3.55
τ	.213	3.56		

Note: $r_{i,t}$ is the excess return of the ith sector, $\eta_{i,t}$ is the idiosyncratic error, and γ_i its conditional variance (time-invariant). $r_{f,t}$ is the excess return of the factor representing portfolio and f_t is the unobservable factor.

B. Unobserved Factor Representing Portfolio

From the previous subsections, it can be claimed that the VW Index CAPM failed in all four hypotheses. One potential solution would be to consider alternative benchmark portfolios, such as the equally-weighted index. Alternatively, the specification of the basis portfolio can be avoided and a model with a single factor can be estimated by maximum likelihood. Here the second route is followed.

Static Latent Factor Model

Assuming the factor and idiosyncratic errors have constant conditional variances, the factor loadings, the factor scores, and the constant idiosyncratic variances can be estimated by maximum likelihood. For identification purposes, the variance of the factor are set to one (see Sentana and Fiorentini [1997]).

Table 7 presents the estimated factor loadings and the associated *t-statistics*. Due to the scaling of the factor, these estimates are completely different from the market betas obtained in section II.D.

TABLE 8. Jensen's α (Static Latent Factor)

$$r_{i,t} = a_i + \beta_i r_{f,t} + \eta_{i,t}$$

$$r_{f,t} = \tau + f_t,\ f_t \sim N(0,1),\ \eta_{i,t} \sim N(0,\gamma_i),\ (i = 1,\ldots,9)$$

Sector	a_i	*t*-statistic
Banks	.716	2
Investment	–.437	–1.16
Insurance	–.19	–.45
Miscellaneous	–.102	–.45
Holding	–.277	–.65
Textile	–.353	–.83
Construction	.459	1.25
Food	.414	1.19
Chemicals	–.09	–.26

Note: $r_{i,t}$ is the excess return of the i^{th} sector, $\eta_{i,t}$ is the idiosyncratic error, and γ_i its conditional variance (time-invariant). $r_{f,t}$ is the excess return of the factor representing portfolio and f_t is the unobservable factor.

However, the correlation between these estimates is .907, fairly high. Notice that this time the price of risk, τ, is significant, at conventional levels (*t-statistic* = 3.68). Jensen's *α's* are presented in table 8. Only the Banks sector has a constant which is significantly different from zero. The joint *LR* test, for all nine constants, equals 10.61, which is insignificant even at the 10% level. It seems that the limiting factor portfolio is close enough to the tangency portfolio to avoid rejection.

The excess returns of the estimated limiting factor portfolio are similar to the VW Index ones, with a correlation of 88.17%. To get a feeling for the common factor, the estimated limiting portfolio excess returns are regressed on the nine sectorial portfolio excess returns and these weights are compared with the coefficients from the regression of the VW Index returns on the nine portfolios. However, notice that since it is assumed that the variances and covariances of asset returns are constant over time, so are the weights, used by the Kalman filter to estimate the factor. Consequently,the R^2 from the first regression will be 1, which is not true for the R^2 of the second one (R^2 =.84), as the VW Index is constructed by all stocks listed in the ASE with time-varying weights. Table 9 presents the coefficients from the two regressions. It

TABLE 9. Average Weights Based on the OLS Regressions of the Factor Estimates and $r_{VW,t}$ on $r_{i,t}$ (i=1,..,9)

Sector	Unobservable Factor	VW Index
Bank	.161	.358
Investment	.175	.100
Insurance	.115	.044
Miscellaneous	.266	.088
Holding	.096	.063
Textile	.136	.045
Construction	.146	.045
Food	.150	.135
Chemicals	.215	.087

TABLE 10. Factor Loadings (Dynamic Latent Factor)

$$r_{i,t} = \beta_i r_{f,t} + \upsilon_{i,t}, (i = 1, \dots, 9)$$

$$r_{f,t} = \tau\lambda_t + f_t,\ \lambda_t = a_0 + a_1 f_{t-1|t-1} + a_2\left(f^2_{t-1|t-1} + \lambda_{t-1|t-1}\right) + a_3\lambda_{t-1}$$

Sector	β_i	t-statistic	$\tau\,\beta_i$	t-statistic
Banks	6.707	6.55	.575	1.95
Investment	4.261	6	.365	1.93
Insurance	6.691	6.03	.573	1.92
Miscellaneous	2.957	6.14	.253	1.93
Holding	5.507	6.36	.472	1.94
Textile	6.364	5.99	.545	1.94
Construction	4.417	6.12	.378	1.91
Food	5.789	6.06	.496	1.92
Chemicals	6.605	6.41	.566	1.93
τ	.085	2.12		

Note: $r_{i,t}$ is the excess return of the ith sector, $\upsilon_{i,t}$ is the idiosyncratic error .$r_{f,t}$ is the excess return of the factor representing portfolio, f_t is the unobservable factor and λ_i is its conditional variance (GQARCH).

can be seen that, whereas the VW Index represents sectors with larger capitalizations, such as Banks, a fact which brings forth the point raised in Adcock and Clark (1999), the weights for the estimated factor are more evenly distributed.

Dynamic Latent Factor Model

Let us now relax the assumption of constant, over time, factor variance and idiosyncratic ones. The maximum likelihood estimates for the betas and the price of risk are presented in table 10. Notice that the β_i's are again different from the ones in the conditional CAPM, due to the fact that the factor is scaled to have a unit-unconditional variance. The correlation between the two sets of β_i's is .76, which is not very close. As in the static setup, the price of risk, τ, not only has the right sign, but is also significant, at least at 5% level (*t-statistic* = 2.12).

There is a dual interpretation of the dynamic latent factor model. It can be interpreted either as a one-latent factor APT model or as a CAPM with an unobserved portfolio proxying the market one. To make this clearer, assume for the moment that the number of factors is $k > 1$. Then, assuming that the market portfolio is well-diversified, the risk premium of any asset i is given by:

$$\mu_{i,t}^{CAMP} = a_t\beta_{M,1,t}\lambda_{1,t}\beta_{i,1,t} + a_t\beta_{M,2,t}\lambda_{2,t}\beta_{i,2,t}$$

$$+\ldots+a_t\beta_{M,k,t}\lambda_{k,t}\beta_{i,k,t}, \tag{7}$$

where

$$a_t = \frac{v_{M,t} - R_{o,t}}{\beta^2_{M,1,t}\lambda_{1,t} + \beta^2_{M,2,t}\lambda_{2,t} + \ldots + \beta^2_{M,k,t}\lambda_{k,t}}$$

independent of the asset i. Comparing now the above expression of asset's i risk premium with the APT risk premium (equation 2), it is concluded that the two expressions are equal if and only if:

$$\frac{\tau_{1,t}}{\beta_{M,1,t}} = \frac{\tau_{2,t}}{\beta_{M,2,t}} = \cdots \frac{\tau_{k,t}}{\beta_{M,k,t}} = a_t, \tag{8}$$

which, for $k > 1$, is in principle a testable hypothesis (see Demos, Sentana, and Shah [1994]). Intuitively, this restriction says that in the CAPM, the factors are only priced through their influence on the market portfolio. It is important to emphasize, though, that the CAPM pricing restrictions are robust to the crucial assumption in the APT that returns

TABLE 11. GQARCH(1,1)-M for the Unobserved Limiting Factor Representing Portfolio

$$\lambda_t = a_0 + a_1 f_{t-1|t-1} + a_2\left(f^2_{t-1|t-1} + \lambda_{t-1|t-1}\right) + a_3\lambda_{t-1}$$

Parameter	Value	t-statistic
a_o	.033	n/a
a_1	.108	3.33
a_2	.232	5.18
a_3	.723	14.09

Note: λ_t is the conditional variance (GQARCH) of the unobservable factor f_t and $f_{t-1|t-1}$ is its Kalman filter estimate.

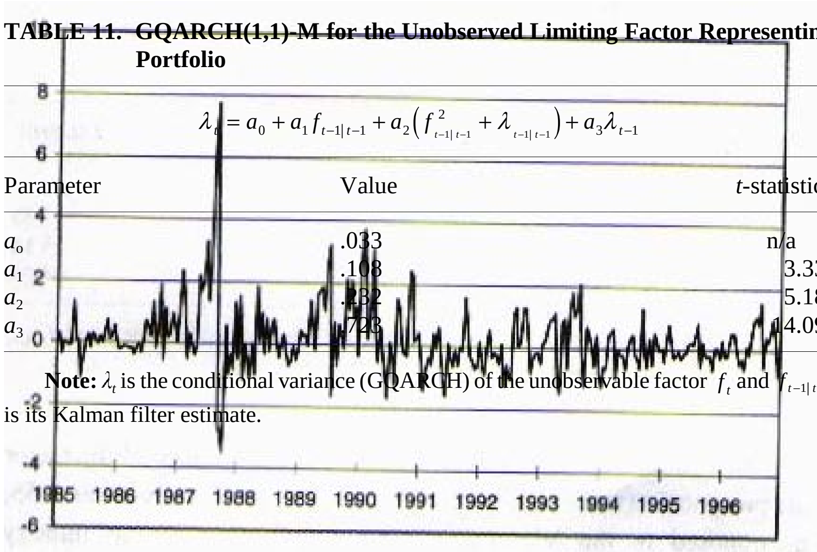

FIGURE 2.—Unobserved factor for a representative portfolio.

follow a conditional factor structure. Therefore, taking into account the assumed time invariance of prices of risk and betas, equation 8 becomes:

, for all k's.

Unfortunately for $k = 1$ the above relationship is trivially true. Hence it is not possible to distinguish between the two models.

Figure 2 presents the estimated excess returns of the limiting factor-representing portfolio. Qualitatively they are similar to the VW Index ones; the correlation between them is .886. There is also a very high correlation between the limiting-factor portfolio of the conditional model and the one from the static (correlation coefficient = .981).However, the conditional factor representing portfolio is more efficient, in an unconditional sense, with a Sharpe Index of .231, as compared to the static one, .213, and to the VW Index, .117.

The estimated parameters in the conditional variance of the factor are presented in table 11. The persistence is now marginally lower, .955, as opposed to the VW Index persistence, .959. The asymmetry parameter, although smaller, is still positive and significant (*LR test* = 5.56). In figure 3, the factor-conditional variance is presented. Notice the increase in the conditional variance during October-November 1987 and the summer of 1990.

In well-developed markets, the persistence parameters, for monthly excess equity returns, range from .58 for Germany to .975 for the U.S.; e.g., De Santis and Gerard (1997). Using the results in Drost and Nijman (1993), the volatility persistence of the monthly returns, implied by the estimates, is .912, a value which is not considered very high. On the other hand, the price of risk reported for various markets range from .023, internationally; in De Santis and Gerard (1997) to .06, for the U.S.; in Ng (1991). Consequently, the estimated value of .085 is considered rather high. Furthermore, a GARCH in Mean specification implies positive auto-correlations for the excess return process (see Fiorentini and Sentana [1998]). These auto-correlations depend on the price of risk and the auto-covariances of the conditional variances. Consequently, the estimated values of these parameters imply higher auto-correlations in the ASE excess returns than the excess returns of various developed markets. Hence, it can be said that as far as the ASE is concerned, the facts documented in Bekaert and Harvey (1997) for emerging market excess returns, namely higher sample average returns,

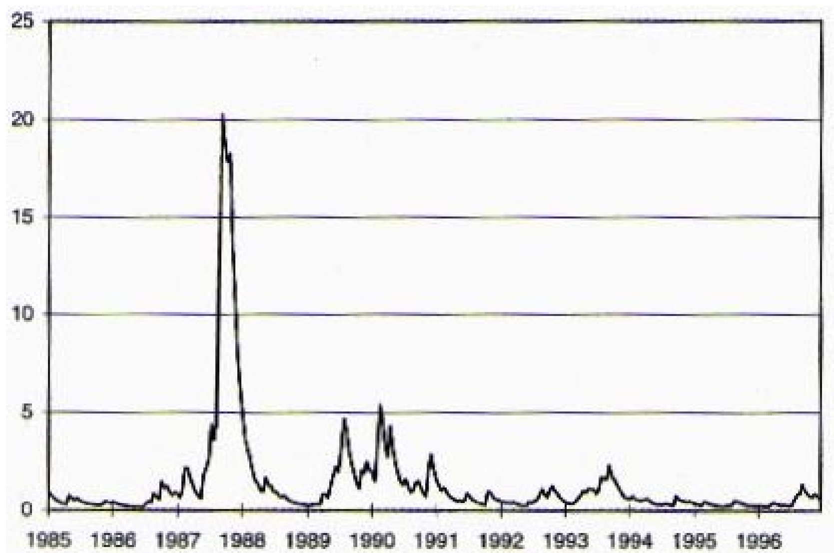

FIGURE 3.—Conditional Factor Variance.

more predictable returns, and higher volatility, comply with a conditionally heteroskedastic one-latent factor asset pricing model.

Out of the 9 idiosyncratic conditional variances 4 are modeled as GQARCH(1,1), Banks, Miscellaneous, Holding, and Food, and the rest as GARCH(1,1). Persistencies range from .487, in Holding, up to .967, in Banks. In terms of the asymmetry parameters, two are negative in Banks and Food, –1.096 and –.550 respectively, and the other two positive, in Miscellaneous and Holding, .533 and 1.716. With the exception of Chemicals, persistencies are lower compared to those of the conditional CAPM.

The tests for the four hypotheses of interest are exhibited in table 12. The hypothesis of zero constants in the mean specifications of the 9 sectorial indices cannot be rejected even at 10% significance level (*LR* = 11.88).This, in fact, means that, on average, the latent factor model prices assets correctly. Furthermore, the hypothesis of common price of risk across assets cannot be rejected at 5% level, but the same does not apply for a significance level of 10% (*LR* = 14.45). Finally, as it is already discussed, having idiosyncratic variances that are time-varying idiosyncratic risk. The *LR* test for the non-idiosyncratic risk-pricing null is 13.13, which is insignificant even at 10% significance level.

Overall, the conditional latent factor model performs quite well. In

TABLE 12. Hypothesis Testing for Sectorial Portfolios (Dynamic Latent Factor Model)

$$r_{i,t} = \beta_i r_{f,t} + \upsilon_{i,t} \quad (i = 1,\dots,9)$$
$$r_{f,t} = \tau\lambda_t + f_t,\ \lambda_t = a_0 + a_1 f_{t-1|t-1} + a_2\left(f^2_{t-1|t-1} + \lambda_{t-1|t-1}\right) + a_3\lambda_{t-1}$$

Hyptohesis	Test Value	Critical Value Under the Null	
Is Risk Priced	4.10	$\chi^2_{1.5\%} = 3.84$	
Non-zero Constants in Mean	11.88	$\chi^2_{9.1\%} = 14.7$	$\chi^2_{9.5\%} = 16.9$
Is the Price of Risk Common	14.45	$\chi^2_{8.1\%} = 13.4$	$\chi^2_{8.5\%} = 15.5$
Is Idiosyncratic Variances Priced	13.13	$\chi^2_{9.1\%} = 14.7$	$\chi^2_{9.5\%} = 16.9$

Note: $r_{i,t}$ is the excess return of the i^{th} sector, $\upsilon_{i,t}$ is the idiosyncratic error . $r_{f,t}$ is the excess return of the factor representing portfolio, f_t is the unobservable factor and λ_t is its conditional variance (GQARCH).

terms of the original questions, the model does very well in three out of four, i.e., non-diversifiable risk is priced, the model prices assets on average correctly, and idiosyncratic risk is not priced. There seems to be a small problem with the price of risk which appears to be not common across the nine portfolios, at least at a high significance level. However, this rejection is not very strong, as it does not exist at a lower level. Nevertheless, taking into account that the model is a relatively simple one, with only one factor, and also the changes that took place in the ASE since 1985, especially since 1989, the model performs overall fairly well.

IV. Conclusion

This article employs an Asset Pricing Model where excess asset returns have a conditional factor representation and, consequently, risk premia can be written as an exact linear combination of the volatility of these common factors. For estimation purposes, it is assumed that the number of factors is one, the factor loadings and risk price are time-invariant, and the factor conditional variance follows an asymmetric, to positive and negative shocks, ARCH-type model. The model is observationally equivalent to a conditional CAPM where the market portfolio is approximated by an unobservable limiting factor-representing portfolio. The Kalman filter is employed to extract this factor. Conditionally on the available information set, i.e., the asset returns under consideration, the Kalman filter estimated factor-representing portfolio is the best approximation, in mean square error terms, to the market portfolio.

Furthermore, depending on the observability of the factor, tests are presented which are based on the three principles of Likelihood Ratio, Wald, and Lagrange Multiplier, to address questions of economic interest such as the integration of the market, the efficiency of the market proxy, and the pricing of diversifiable and non-diversifiable risk. Specifically, by modelling the idiosyncratic conditional variances as ARCH-type models, the hypotheses that the model prices assets correctly, on average, can be distinguished from the pricing of idiosyncratic risk. From a methodological point of view, it turns out that, under the assumption that a conditionally homoskedastic base portfolio is observed, all three tests are easily computed. However, alternatively, if the benchmark portfolio is unobserved and is conditionally heteroskedastic, the application of the Likelihood Ratio test is by far easier to compute than the Wald or the Lagrange Multiplier, which is the most difficult of the three to apply.

The above tests are applied to fortnightly returns of the nine ASE sectorial indices and the Value Weighted Index. The data rejects unconditional and conditional versions of CAPM where the benchmark portfolio is the Value Weighted Index, something which is in agreement with a considerable number of studies that reject the CAPM with an observable benchmark portfolio, in a variety of countries; e.g., Sentana (1997), for Spain, De Santis and Gerard (1997), Ng (1991), Hall et al. (1989). In fact, this model failed in all four asset pricing restrictions, i.e. it does not price, on average, assets correctly, the non-diversifiable risk is not priced, the price of risk is not common across assets, and diversifiable risk is priced. However, a latent factor model with GQARCH conditional variance and ARCH-type idiosyncratic ones is not rejected. In terms of the MacKinlay (1995) terminology, the VW Index CAPM is rejected not because of missing additional risk factors, as in Bekaert and Harvey (1997) where an additional world factor is considered, but mainly due to the fact that the benchmark portfolio fails to approximate the market adequately. This point becomes apparent when it is revealed that the Sharpe Index of the latent factor limiting portfolio is twice as much as that of the VW Index.

A fact that also emerges from the empirical application is that the asymmetry effect is the opposite from what is observed in the developed, U.K. and U.S., markets, i.e., positive unexpected returns have a higher impact on volatility than negative ones of the same size. This is consistent first with previous studies of the ASE, and it seems that it does not depend in the specific GARCH parameterization (see Koutmos, Negakis, and Theodossiou (1993), where an Exponential GARCH model is employed) and, second, with studies of another emerging market such as the Spanish Stock Exchange; e.g., Sentana (1997). However, whether this fact can be generalized as to apply to other emerging markets is still open to question.[4] Finally, as far as the ASE is concerned, the facts documented in Bekaert and Harvey (1997) for emerging market excess returns, namely higher sample average returns, more predictable returns, and higher volatility, comply with a conditionally heteroskedastic one-latent factor asset pricing model.

Of course not only the restrictions of interest are tested, but the maintained assumptions that underlie the intertemporal asset pricing model and its empirical implementation as well. Specifically, the assumptions of constant factor loadings and constant price of risk are potentially restrictive. Furthermore, various other stylized facts of stock market returns, such as seasonalities, were not incorporated. Nevertheless, the model stands all the tests under consideration fairly well, so that it can be concluded that the sectorial ASE stock returns comply to a time varying volatility latent factor model. This result has important implications for international and local investors in terms of evaluating direct investment and asset allocation decisions.

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4. Notice that Bekaert and Harvey (1997) use a symmetric conditional variance parameterization, and consequently, they were not able to investigate this issue.

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