Dynamic Autocorrelation and International Portfolio Allocation

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We explore the relevance of dynamic autocorrelation in modeling expected returns and allocating funds between developed and emerging stock markets. Using stock market data for the US and Latin America, we find that autocorrelation in monthly returns vary with conditional volatility, implying some investors implement feedback trading strategies. Dynamic autocorrelation models fit the data considerably better than a conditional version of the zero-beta CAPM, while differences between models with an autoregressive term are modest. Investors can improve their portfolio optimization between developed and emerging stock markets by considering time-varying autocorrelation. The most drastic difference in portfolio performance is not due to allowing autocorrelation to vary over time, but realizing that stock returns are autocorrelated, especially in emerging stock markets. (JEL: G11, G12, G15)

Keywords: autocorrelation; volatility; portfolio; international; emerging markets

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I. Introduction

The intertemporal trade-off between expected return and risk (Merton, 1973) has attracted enormous interest in the empirical finance literature (see Brand and Wang, 2010; Nyberg, 2012, Ghysels et al., 2014). The contemporaneous role of dynamic autocorrelation in driving expected returns has received less attention, especially in practical applications. In this study, we explore the relevance of the risk-return trade-off and time-varying autocorrelation in modeling expected returns and allocating funds between developed and emerging stock markets.

The traditional view on market efficiency says past returns should not contain relevant pricing information if the asset-pricing model is correct and the financial market is efficient. In practice, past returns often help to forecast stock returns along a conditional risk-return trade-off (Bollerslev et al., 1988; Ghysels et al., 2005; Kinnunen 2014). Heterogeneous agent explanations for autocorrelation include the Sentana-Wadhwani (SW) feedback trading model (Sentana and Wadhwani, 1992). In the presence of investors whose demand for shares is based on past price changes, both a conditional risk-return trade-off and autocorrelation that varies with volatility can cause predictability in returns.¹ Sentana and Wadhwani (1992) and Koutmos (1997a) report empirical support for the SW model in developed stock markets, whereas Koutmos and Saidi (2001) find similar support in emerging stock markets. Models that incorporate feedback trading have been also studied by Cutler et al. (1990) and DeLong et al. (1990), among others. We use the SW model as the benchmark autocorrelation model. As an alternative model, we consider the exponential autoregressive model with volatility of LeBaron (1992). This model has been applied by Chen et al. (2008) and Koutmos (1997b) to investigate autocorrelation patterns in the US and Asian stock returns, respectively.

Earlier studies do not compare the performance of the SW model against traditional asset-pricing models or alternative models with time-varying autocorrelation. This would be essential if one wishes to evaluate how well the SW model and other dynamic autocorrelation models perform in modeling expected stock returns. Moreover, there are various explanations for autocorrelation in stock returns (see Campbell et al. 1997).

¹ There are various explanations for autocorrelation in stock returns (for an overview, see Campbell et al. 1997). Explanations range from spurious autocorrelation caused by nonsynchronous trading (see Lo and MacKinlay, 1990) to autocorrelation caused by an interplay between conditional mean and variance process (Hong, 1991).
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These explanations range from spurious autocorrelation caused by nonsynchronous trading (Lo and MacKinlay, 1990) to autocorrelation caused by an interplay between conditional mean and variance process (Hong, 1991). Since most studies investigate dynamic autocorrelation patterns using hourly, daily or weekly data, it is difficult to distinguish between autocorrelation induced by nonsynchronous trading and autocorrelation due to heterogeneous agents and other economic explanations. In addition, the relevance of time-varying autocorrelation in empirical applications such as portfolio optimization also remains unexplored.

This study has three overarching goals. First, we examine simultaneously the effect of conditional volatility on autocorrelation patterns of the US and Latin American stock market returns. Autocorrelation in monthly returns is found to vary with volatility as suggested by the SW model. This comports with the findings of Sentana and Wadhwani (1992), Koutmos (1997a), Koutmos and Saidi (2001), Bohl and Siklos (2008), and Kinnunen (2014), who report similar behavior in daily returns for both emerging and developed stock markets. As we employ monthly returns, the finding strengthens the view that autocorrelation is caused by heterogeneous agents rather than nonsynchronous trading, which can cause spurious autocorrelation in daily returns but is unlikely to be a serious problem in monthly stock index returns. Harvey (1995) notes the serial correlation observed in emerging market returns is usually higher than that found in developed markets. Our results agree with this view: time-varying autocorrelation in the Latin America stock portfolio return is usually higher than in the US aggregate market return. This finding agrees with the results reported by Kinnunen (2013; 2014), among others.

We also extend earlier analyses by controlling our results against the influence of global factors. Autocorrelation in unadjusted returns may reflect time-varying risk premia (Anderson, 2011). Previous autocorrelation studies seldom control their results for increased financial integration between world capital markets. Roll and Pukthuanthong (2009), however, report evidence on increased market integration, implying that the earlier results on dynamic autocorrelation

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2. DeSantis and Imrohoroglu (1997) find support for a regional and global risk-return relation in Latin America, while simultaneously reporting significant time-invariant autocorrelations in stock index returns and rejecting full segmentation at country-level. Thus, in addition to its size and colorful economic history, Latin America appears to be an ideal emerging market area to investigate simultaneously the risk-return trade-off and time-varying autocorrelation in returns.
may be partly driven by omitted global pricing factors. We find that
global stock market return and changes in the oil price have little
explanatory power for expected returns in addition to a local risk-return
trade-off and a first-lag autoregressive term that varies with volatility.

Second, the performance of the SW model in explaining expected
returns is evaluated against the zero-beta CAPM and the exponential
autoregressive model with volatility proposed by LeBaron (1992). The
latter model can be economically motivated by the adaptive market
hypothesis of Lo (2004). The SW model performs considerably better
than a conditional version of the zero-beta CAPM. This suggests that
heterogeneous agent models offer a more realistic representation of
stock returns than models assuming a single representative investor.3
However, differences in performance between models with a first-order
autoregressive term are modest. The gain from accounting for
time-varying autocorrelation on a model’s performance in explaining
monthly returns is small. This result is related, at least partly, to the
monthly return interval used. The long interval makes it easier to
distinguish nonsynchronous trading from feedback trading, but weakens
the level of time-varying autocorrelation in returns.

Third, the relevance of dynamic return autocorrelation in
international portfolio optimization is studied. Previous authors have
found, for example, that intertemporal hedging demands as well as
regime-switches in financial ratios and market conditions can be useful
in asset allocation (Brooks and Persand, 2001; Ang and Bekaert, 2002;
Gerard and Wu, 2006). Recently, DeMiguel et al. (2014) use a
vector-autoregressive (VAR) model and find that taking serial
dependence in portfolio-level US returns into account can improve
portfolio performance. We find that noticing time-varying
autocorrelation in modeling expected returns can improve an investor’s
portfolio allocation between a developed and emerging stock market in
the cases of a global minimum variance portfolio and a tangency
portfolio when a risk-free asset exists and short sales are restricted. The
first case measures the indirect effect of time-varying autocorrelation
via forecasting errors’ effect on the conditional covariance matrix. The
second case additionally measures the effect of dynamic autocorrelation
that comes directly through expected returns. Ignoring autocorrelation
completely appears to cause a considerable underperformance of a

3. For a survey on heterogeneous models in economics and finance, see Hommes (2006)
and Manzan (2009).
portfolio. It may also lead to over-investment in an emerging market.

The rest of this study is organized as follows. In Section II, we discuss the SW model and the exponential autoregressive model with volatility. Section III contains data description. Section IV presents the empirical results, robustness tests against global factors, and portfolio performance evaluations. The final section summarizes.

II. Autoregressive models with volatility

A. Feedback trading model

In the SW model (Sentana and Wadhwani, 1992), feedback trading causes the serial correlation in stock returns. The model is based on a heterogeneous agent assumption with two groups of investors (see also Shiller, 1984). The first investor group’s demand for shares is based on risk-return considerations. Let \( E_{t-1}[R_t] \) and \( \sigma_t^2 \) denote the conditionally expected return and its conditional variance at time \( t \), both conditional on the information set available to investors at time \( t-1 \). The fraction of shares demanded by the first group is given as \( Q_1 = (E_{t-1}[R_t] - \alpha) / \lambda \sigma_t^2 \), where \( \alpha \) is the return at which the demand is zero. The denominator is the risk premium required by the first group of investors to hold all shares. Here, we assume that the first group is risk averse, so \( \lambda > 0 \). If the expected return increases or the required risk premium decreases, the first group demands a greater portion of all shares.

The second group of investors exercises feedback trading. Their demand is given as \( Q_2 = \gamma R_{t-1} \). A positive feedback strategy \( (\gamma > 0) \) means that investors buy (sell) after price increases (decreases). As discussed by Sentana and Wadhwani (1992), such behavior agrees with portfolio insurers and those using stop-loss orders. In contrast, a negative feedback strategy \( (\gamma < 0) \) implies that investors buy after price declines, which is consistent with the behavior of those investors following ‘buy low/sell high’ strategies. In equilibrium, all shares are

4. Autocorrelation in returns can have important implications in various financial applications such as beta estimations (Scholes and Williams, 1977) and derivatives pricing (Jokivuolle, 1998).

5. Some investors may follow positive feedback trading strategies and some implement negative feedback trading. Because \( \gamma \) incorporates the impact of both positive and negative feedback traders, \( \gamma > 0 \) \( (\gamma < 0) \) implies that the impact of positive (negative) feedback trading
held and it holds that $Q_t^1 + Q_t^2 = 1$. This yields the SW model:

$$E_{t-1}[R_t] - \alpha = \lambda \sigma_t^2 - \gamma \lambda \sigma_t^2 R_{t-1}. \quad (1)$$

The last term on the right side implies that returns will exhibit negative (positive) autocorrelation if the second investor group follows a positive (negative) feedback trading strategy. The autoregressive term varies with conditional volatility. Expected return in eq. (1) can be time-varying and predictable due to the time-varying risk premium and/or because of the first-order autoregressive component. We use eq. (1) as the benchmark autocorrelation model.

If the first group demand all shares ($Q_{t1}^1 = 1$) and $\alpha$ is set equal to the risk-free rate, the model yields a discrete-time approximation of the intertemporal CAPM (ICAPM) of Merton (1973; 1980). If the risk-free asset does not exist, eq. (1) without the time-varying autoregressive term can be interpreted as a conditional version of Black’s (1972) zero-beta CAPM, which we use as the benchmark asset-pricing model. As the second traditional model, we consider the conditional zero-beta CAPM with a constant first-order autoregressive coefficient. In the asset-pricing literature, a constant autoregressive term along the risk-return trade-off is usually motivated by nonsynchronous trading (e.g., Nelson, 1991; DeSantis and Imrohoroglu, 1997) or as a test of whether lagged returns have predictability ability along a conditional asset-pricing model (e.g., Bollerslev et al. 1988; Ghysels et al., 2005). Somewhat surprisingly, asset-pricing studies often completely ignore the potential explanatory power of lagged returns.

After replacing the conditionally expected return by the realized return and an error term, an empirical bivariate version of eq. (1) can be tested by the following system of equations

$$R_{US,t} = \alpha_{US} + \lambda_{US} h_{USUS,t} + \left( \rho_{0US} + \rho_{USUS,t} h_{USUS,t} \right) R_{US,t-1} + e_{US,t} \quad (2)$$

$$R_{LA,t} = \alpha_{LA} + \lambda_{LA} h_{LALAL,t} + \left( \rho_{0LA} + \rho_{LALAL,t} h_{LALAL,t} \right) R_{LA,t-1} + e_{LA,t}$$

where $US =$ United States, $LA =$ Latin America, and $h_{ii,t}$ is the $ii$th element of the 2 x 2 conditional covariance matrix of the error terms $H_t$. Following Merton’s (1980) suggestions, we model risk-return coefficients in eq. (2) as an exponential function $\lambda_i = \exp(\delta_i)$. This

is stronger than the impact of negative (positive) feedback trading.
restricts the risk premium demanded by the first group of investors to be positive. In the language of international asset pricing, eq. (2) assumes a country-level risk-return trade-off for the US stock market and a regional risk-return trade-off for the Latin American stock market.6

B. Exponential autoregressive model with volatility

As an alternative to eq. (2), we consider the work of LeBaron (1992), who allows the first-order autocorrelation to vary using the exponential autoregressive model with volatility:

\[ R_{US,t} = \alpha_{US} + [\rho_{US} + \rho_{US} \exp(-h_{US,t}/b_{US})]R_{US,t-1} + \varepsilon_{US,t} \]

\[ R_{LA,t} = \alpha_{LA} + [\rho_{LA} + \rho_{LA} \exp(-h_{LA,t}/b_{LA})]R_{LA,t-1} + \varepsilon_{LA,t} \]

(3)

The second term in brackets induces time variation in the first-order autoregressive coefficient. Following LeBaron (1992), in empirical estimations, \( h_i \) are set equal to sample variances of returns to avoid problems with numerical optimization. During high variance periods, the first-order autoregressive coefficient is given mainly by \( \rho_{0i} \). Notably, in theory, if \( h_{0i}/b_i \to \infty \), an equation for series \( i \) reduces to an autoregressive model of order 1. During periods of low variance, the first-order autocorrelation is close to \( \rho_{0i} + \rho_i \).

Eq. (3) can be economically motivated by the adaptive market hypothesis of Lo (2004). The traditional view on market efficiency implies that if markets are weak-form efficient, predictability using historical prices should be at an economically insignificant level (Fama, 1970). Under the AMH, investors constantly adapt to changing market conditions with satisfactory (rather than optimal) behavior, which can cause dynamic return predictability.7 Kim et al. (2011) analyze the US stock market and find empirically support for the AMH. Since volatility reflects changes in market conditions, eq. (3) can capture time-varying return predictability in the way that the AMH indicates it should manifest itself. Specifically, returns can be predictable under certain market conditions, while unpredictable otherwise. Kinnunen (2014) discusses this idea in detail.


7. Note that the ICAPM also allows dynamic return predictability due to changing market conditions.
The innovation vector $\varepsilon_t = [\varepsilon_{US,t}, \varepsilon_{LA,t}]'$ is assumed to be conditionally normally distributed,

$$\varepsilon_t | Z_{t-1} \sim N(0, H_t).$$

(4)

The conditional covariance matrix $H_t$ follows the BEKK model of Engle and Kroner (1995):

$$H_t = C'C + A'\varepsilon_{t-1}^2A + B'H_{t-1}B,$$

(5)

where $C$ is a 2 x 2 upper triangular matrix, and $A$ and $B$ are 2 x 2 parameter matrices. Asset-pricing models such as the conditional CAPM do not impose any restrictions on the dynamics of the conditional second moments. The parameterization (5) reflects recommendations of Bollerslev et al. (1988), who state based on their results that any correctly specified intertemporal asset-pricing model should account for the heteroscedastic nature of asset returns. More precisely, GARCH models are able to capture some typical features found in financial data, e.g. leptokurtic distribution and volatility clustering.

All parameters are estimated simultaneously using maximum likelihood. Since financial time series often violate the normality assumption, we follow standard practice and estimate all models using the quasi-maximum likelihood (QML) approach of Bollerslev and Wooldridge (1992). Statistical inferences are made using robust QML standard errors and Wald statistics.

### III. Data

Monthly returns on the US and Latin America aggregate stock market
portfolios are used as the test assets. The data for Latin America area has several useful features: it includes Brazil (BRIC country) and the region has undergone several major economic events (e.g. the Argentine economic crisis that began in 1999). The latter feature implies that the return generating process has changed over time, suggesting that dynamic autocorrelation models may fit the data better than traditional models. Moreover, DeSantis and Imrohoroglu (1997) find support for both regional and global risk-return trade-off in the Latin American stock market, while simultaneously reporting significant time-invariant autocorrelations in stock index returns and rejecting full segmentation at country-level. Thus, Latin America should nicely demonstrate the relevance of the risk-return trade-off and dynamic return autocorrelation and the potential effect of omitted global factors.

We proxy the stock market performance using the Thomson Datastream equity market indices with dividends reinvested. The indices are value-weighted. All returns are expressed in US dollars and percentage form. The sample period is from January 1995 to January 2012 (205 observations). Financial markets in many Latin America countries opened for foreign investors around 1990, implying that the sample covers the period under which regional integration is a plausible assumption.11

Table 1 presents descriptive statistics. Monthly mean return for the US stock portfolio is 0.821%, whereas the Latin America stock portfolio has earned on average 1.101% per month. Standard deviations are 4.959% and 7.968% for the US and Latin America return series, respectively. As one would expect, the emerging stock market offered a higher average return than the developed stock market, but the returns simultaneously carry increased uncertainty (measured by a higher standard deviation). Both series exhibit significant negative skewness and positive excess kurtosis. In both cases, the Jarque-Bera test rejects the null of unconditional normality, indicating that the QML approach is appropriate.

The Latin America stock return series exhibit positive first-lag autocorrelation, whereas all autocorrelation coefficients for the US return series are statistically insignificant. This finding agrees with Harvey (1995), who mentions that autocorrelations in emerging markets

11. For more details, see DeSantis and Imrohoroglu (1997). The DS Latin America equity index is available from July 1994. The DS Latin America equity index includes the following countries: Brazil, Argentina, Colombia, Mexico, Chile, Venezuela, and Peru. In terms of nominal gross-domestic product, Brazil is the largest economy in Latin America. The size of economy and the levels of economic complexity vary across countries in Latin America.
can be expected to be higher than those observed in developed stock markets. Engle’s (1982) Lagrange multiplier (LM) test of order 12 shows that the ARCH effects are present. Therefore, the GARCH models are appropriate for the data.

### IV. Empirical Results

#### A. Model estimates

Table 2 shows QML estimates of the mean parameters for the SW model and its alternatives. The lower part of the table shows robust Wald statistics, information on model fit, and diagnostic statistics on standardized residuals. For every specification, the first column shows estimates for the US return series. The second column shows corresponding results for the Latin America (LA) return series. The conditional zero-beta CAPM with and without the time-invariant first-order autoregressive term are both restricted forms of the SW model. In the first case, $\rho_{1US} = \rho_{1LA} = 0$. The second case results if $\rho_{1US} = \rho_{1LA} = \rho_{0US} = \rho_{0LA} = 0$.

For the SW model, the obtained point estimates for $\rho_0$s are
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significant, positive, and considerable in magnitude. In addition, the point estimates for the parameters that allow return autocorrelation to vary with volatility are statistically significant and negative for both series. This implies that the level of the first-order autocorrelation is negatively related to volatility. These conclusions are further supported by the Wald statistics. The joint null hypotheses of zero autoregressive coefficients and time-invariant autocorrelation are both rejected. The evidence on the presence of dynamic autocorrelation agrees with Sentana and Wadhwani (1992) and Koutmos (1997a).

Figure 1 visualizes the estimated time-varying first-order autoregressive coefficients obtained using the SW model. The plot reveals that autocorrelation in returns on the US and Latin America aggregate stock portfolios are clearly time-varying and mostly positive. The level of the estimated first-order autoregressive coefficients is generally higher for the Latin America return series. This agrees with Harvey (1995), who mentions that the serial correlation observed in emerging market returns is higher than that found in developed markets. The finding also comports with the empirical results of Bohl and Siklos (2008) and Kinnunen (2013; 2014).

Since in eq. (1) is modeled as \( \gamma \lambda \sigma^2 \) and the risk-return coefficient \( \lambda > 0 \), the plot shows that to produce the estimated positive autoregressive coefficients feedback traders must mostly follow a negative feedback trading strategy (\( \gamma < 0 \)) in both markets. Negative feedback trading behavior suggests investors are following ‘buy low/sell high’ strategies. Interestingly, around 2009, when financial markets were in volatile stage due to the global financial crisis, the estimated autoregressive coefficients switch to negative. This indicates that investors started to follow a positive feedback strategy (\( \gamma > 0 \)), i.e. behavior typical of portfolio insurers and those using stop-loss orders. In other words, feedback traders switched their trading strategy during the volatile aftermath of the recent global financial crisis. In the empirical formulation, the negative estimates for \( \rho_{1t} \) allow positive feedback trading to offset negative feedback trading during high volatility periods. Note that during these periods, the risk premium demanded by smart money investors increases and simultaneously lowers their demand for shares.

Based on the Akaike information criteria, the time-varying autoregressive component of the SW model does not significantly affect the performance of the model: the conditional zero-beta CAPM with the time-invariant first-order autoregressive term performs nearly as well as
TABLE 2. Quasi-maximum likelihood estimates

<table>
<thead>
<tr>
<th>Parameter estimates</th>
<th>SW model</th>
<th>Exponential AR(1)</th>
<th>Zero-beta CAPM</th>
<th>Zero-beta with AR(1)</th>
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<tr>
<td></td>
<td>US</td>
<td>LA</td>
<td>US</td>
<td>LA</td>
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<tr>
<td>( \alpha_i )</td>
<td>0.753*</td>
<td>0.686</td>
<td>1.049**</td>
<td>1.287*</td>
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<td></td>
<td>(0.328)</td>
<td>(0.637)</td>
<td>(0.262)</td>
<td>(0.489)</td>
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<tr>
<td>( \delta_i )</td>
<td>-4.245**</td>
<td>-4.532**</td>
<td>-4.985</td>
<td>-4.953</td>
</tr>
<tr>
<td></td>
<td>(0.674)</td>
<td>(1.048)</td>
<td></td>
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</tr>
<tr>
<td>( \rho_{0i} )</td>
<td>0.196*</td>
<td>0.323**</td>
<td>0.062</td>
<td>0.036</td>
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<tr>
<td></td>
<td>(0.083)</td>
<td>(0.078)</td>
<td>(0.110)</td>
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<td>( \rho_{1i} )</td>
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<td>-0.002*</td>
<td>0.075</td>
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<td>(0.001)</td>
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<td>( H_0: ) Zero-CAPM*</td>
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<td>( H_0: ) Zero-CAPMb</td>
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<td>( H_0: ) Diagonal B d</td>
<td>78.076**</td>
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(Continued)
### TABLE 2. (Continued)

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<td>Skewness $z$</td>
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<td>-0.337</td>
<td>-0.518</td>
<td>-0.389</td>
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<td>J-B $p$-value $z^f$</td>
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</table>

**Note:** This table shows quasi-maximum likelihood (QML) estimates for the mean parameters of the SW model [Eqs. (2), (4), and (5)] and the exponential autoregressive model with volatility [eqs. (3)-(5)]. The zero-beta CAPM results when all autoregressive coefficients are set to zero in the SW model. In the zero-beta CAPM with AR(1), only the coefficients that allow autoregressive coefficients to vary with volatility are restricted to zero. The GARCH parameter estimates are not reported. For every model, the first (second) column shows results for the US (Latin America) return series. The sample is from January 1995 to January 2012 (205 observations). Robust QML standard errors in parenthesis. a The robust Wald test for the null of conditional zero-beta CAPM ($H_0: \rho_{\text{US}} = \rho_{\text{LA}} = 0$). b The robust Wald test for the null of conditional zero-beta CAPM ($H_0: \rho_{\text{US}} = \rho_{\text{LA}} = 0$). c The robust Wald test for the null of time-invariant autoregressive coefficients ($H_0: \rho_{\text{US}} = \rho_{\text{LA}} = 0$). d The robust Wald test for the null of diagonality restriction on a parameter matrix. e $p$-values for the Jarque-Bera test statistic for normality. f $p$-values for Engle’s (1982) Lagrange multiplier test statistic of order 12 for ARCH effects. ** and * denote significance at the levels of 1% and 5%, respectively.
FIGURE 1.— Time-varying autoregressive coefficients

Note: The figure shows plots of the estimated time-varying autoregressive coefficients for the US and Latin America return series. Estimates are obtained using the SW model [Eqs. (2), (4), and (5)]. The sample period is from January 1995 to January 2012.

the SW model. Notably, if $\gamma$ in reality varies inversely with volatility, it is possible that $-\gamma \lambda \sigma_i^2 = \rho_0$. In this case, the SW model is equal to the zero-beta CAPM with the (time-invariant) first-order autoregressive term. Alternatively, the same result could be due to time variation in $\gamma$ or in both coefficients. However, we do not try to interpret the zero-beta CAPM with the autoregressive term as a special case of the SW model of which underlying prediction is that autocorrelation varies with volatility.

Ignoring the first-order autoregressive component results in considerable underperformance. This is demonstrated by the AIC value of the pure zero-beta CAPM. The results suggest it is important to consider autocorrelation in returns in empirical asset-pricing models. In general, ignoring autocorrelation in returns is likely to affect any application that utilizes estimates for expected stock returns. For instance, obtained estimates for $\delta_i$ differ in their statistical significance between the zero-beta CAPM with and without the autoregressive term, whereas the SW model and the zero-beta CAPM with the autoregressive term agree with their results.

The exponential autoregressive model with volatility performs as well as the two other models with an autoregressive term based on the
AIC value. In this respect, the relevance of the risk-return trade-off component on a model’s performance seems to be small. With the exponential autoregressive model, the point estimates for $\rho_i$'s are all positive, implying that the level of the first-order autocorrelation increases during low-variance periods. This agrees with the results obtained with the SW model. However, with the exponential AR, all estimated autoregressive coefficients are statistically insignificant.

The estimated ARCH and GARCH parameters (untabulated) show that the conditional covariance matrix is clearly time-varying. Based on Engle’s LM test statistics, ARCH effects in standardized squared residuals have disappeared, indicating that the variance parameterizations are satisfactory. The null hypotheses of a diagonal ARCH and GARCH coefficient matrix are both rejected by the robust Wald statistics. Thus, the volatility spillovers between the return series are important in fully capturing the dynamics of the conditional second moments.\(^{12}\)

B. Specification test for global factors

Autocorrelation in unadjusted stock returns may reflect time-varying risk premia (Anderson, 2011). International asset-pricing models (e.g. Errunza and Losq, 1985) predict that if a country’s financial market is financially integrated with world capital markets, the global market risk should be a more relevant pricing factor than the local market risk. For example, DeSantis and Imrohoroglu (1997) find support for both regional and global integration in Latin America. Therefore, it is possible that a global stock market factor could have explanatory power for the expected returns on the stock indices in addition to the SW model and other models tested in the previous section. Previous autocorrelation studies seldom control their results for increased financial integration between world capital markets.

Another potentially relevant global factor is the oil price, which closely reflects developments in the global economy. Changes in the oil price may proxy for marginal utility growth, in which case fluctuations

\(^{12}\) A diagonality restriction for A and B matrices is often imposed in asset-pricing studies to ease numerical estimation of multivariate GARCH-in-mean models (see, e.g., DeSantis and Gerard, 1997; 1998). The above result implies that this may be an overly restrictive practice. More precisely, as González-Rivera (1996) discuss, a rich parameterization that allows interactions between series is desired as the consistency of the GARCH-in-mean parameter depends on a well-specified volatility process.
in the oil price could influence security prices. Thus, oil price changes can have additional explanatory power for the expected returns. Basher and Sadorsky (2006) report that oil-price risk influences stock returns in various emerging markets. For the United States, Elder and Serletis (2010) note that the oil price uncertainty has a negative effect on real output. Hence, it is possible that omitted factors such as oil-price risk and the global market factor are partly influencing the obtained results.

To test whether global factors are influencing the expected returns, we estimate all models with the lagged percentage changes in the Brent oil price and global stock market performance as additional regressors.13

The alternative system of mean equations, for example, for the SW model is

$$
R_{US,t} = \alpha_{US} + \lambda_{US} h_{US,t} + \left( \rho_{US} + \rho_{US} h_{US,t} \right) R_{US,t-1} + \theta_{US} X_{t-1} + \epsilon_{US,t}
$$

$$
R_{LA,t} = \alpha_{LA} + \lambda_{LA} h_{LA,t} + \left( \rho_{LA} + \rho_{LA} h_{LA,t} \right) R_{LA,t-1} + \theta_{LA} X_{t-1} + \epsilon_{LA,t}
$$

where $X_{t-1}$ is a vector of global variables known at time $t-1$ and $\theta$ is a vector of coefficients. Ghysels et al. (2005) and Kinnunen (2014) take

---

13. The global stock market performance is approximated by the Thomson Datastream Global equity index.
a similar approach in their study.

Table 3 shows the robust Wald statistics for the joint null of no additional forecasting power. The conditional zero-beta CAPM is the only model for which the additional global component, $\theta_i X_{t-1}$, is statistically significant. None of the individual coefficients in $\theta_i$ (untabulated) are significant at the 5% level with the exception of the oil coefficient for the US return series with the conditional zero-beta CAPM. This is the only model for which the AIC value is better than without the global factors. For the other models, specifications without the additional component perform better. In general, it seems that the tested global factors do not have additional explanatory power for the expected returns when autocorrelation is taken into account.

C. International portfolio allocation

The previous section shows that the SW model and the exponential AR model fit the data better than the conditional zero-beta CAPM without the time-invariant autoregressive term. The results imply that ignoring autocorrelation in returns can influence any application that uses estimates for expected stock returns. To assess further the economic significance of time-varying autocorrelation in returns, we next analyze its effect on portfolio allocation between the US and Latin American stock markets. To avoid making further assumptions on the level of risk aversion, we first consider the global minimum variance portfolio.14 Second, we analyze a tangency portfolio with a risk-free asset and a shorting constraint.15

Global minimum variance portfolio

For any investor, regardless of their risk aversion, mean-variance optimization implies the following portfolio allocation for the global minimum variance portfolio:

14. More formally, an investor’s mean-variance optimization problem is: $\text{Min } Var(R_p) = \text{Min } w' \Sigma w$ subject to $\sum w = 1$ and $\mu w = \mu$. If a risk-free asset does not exist, the optimal combination of the risky assets depends on the expected rate of return $\mu$, which in turn depends on an investor’s risk preferences. The global minimum portfolio has the lowest variance among all portfolios consisting of risky assets, so its solution does not require setting $\mu$.

15. For more details on portfolio optimization, see Huang and Litzenberger (1988).
\[ w_{\text{MVP}} = \frac{H^{-1}1}{1'H^{-1}1} \]  

(7)

where \( w_{\text{MVP}} \) is the 2 \( \times \) 1 vector of optimal weights for the US and Latin America and 1 is a vector of ones. The global minimum variance portfolio has the lowest variance among all portfolios consisting of only risky assets. As eq. (7) does not include expected returns, the minimum variance portfolio solution nicely demonstrate the indirect effect of time-varying autocorrelation that comes via forecasting errors’ influence on the covariance matrix [modeled using eq. (5)]. More precisely, empirical mean models yield different expected returns, but the actual effect on the allocation decision and realized portfolio performance is caused by forecasting errors’ influence on expected (co)variances.

Estimations in the previous section yield expected returns and (co)variances of returns on the US and Latin America stock portfolios. We track the performance of different models as follows. First, at the beginning of each month, the minimum variance portfolio is chosen based on estimated expected values from different models. Second, the performance of different models is compared based on average realizations over the entire sample period. Gerard and Wu (2006) use the same strategy in analyzing intertemporal risk and its effect on asset allocation.

Table 4 shows average performance characteristics for different models. Since the minimum variance portfolio solution [eq. (7)] does not include expected returns, it is hardly surprising that, on average, the fractions invested in the US and Latin America portfolios do not differ considerably among models. The realized returns and standard deviations, on the other hand, differ more significantly. Based on average returns and standardized average returns, the SW model outperforms the exponential AR and the zero-beta CAPM with or without the first-lag autoregressive term. The SW model earns on average 0.823% per month, whereas worst-performing pure zero-beta CAPM portfolio earns only 0.724% per month. The resulting percentage different in average return and standardized return between these two models is approximately 15%, which is a high figure, especially at the annual level. This translates to a 1.19 percentage point difference per annum, which is further achieved with a lower standard deviation. In fact, the difference between the performance of the model without an autoregressive term and the other models is always substantial, indicating that the first-order autocorrelation should be noted in portfolio allocation decisions.
Figure 2 shows the mean-variance frontier for all static combinations of the US and Latin America weightings, with the minimum variance portfolio realizations for the SW model and its alternatives. The SW model is the only model that beats the static minimum variance portfolio. The average realized return (standard deviation) on the static minimum variance portfolio is 0.8% (4.927%) per month. The results imply that it is important to consider time-varying autocorrelation in portfolio decisions, even if the expected returns are not needed directly as is the global minimum variance portfolio solution. Since the

<table>
<thead>
<tr>
<th></th>
<th>SW model</th>
<th>Exponential AR</th>
<th>Zero-beta CAPM</th>
<th>Zero-beta CAPM with AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction in US stock portfolio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.109</td>
<td>1.100</td>
<td>1.103</td>
<td>1.095</td>
</tr>
<tr>
<td>SD</td>
<td>0.197</td>
<td>0.170</td>
<td>0.166</td>
<td>0.182</td>
</tr>
<tr>
<td>Fraction in Latin America stock portfolio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>−0.109</td>
<td>−0.100</td>
<td>−0.103</td>
<td>−0.095</td>
</tr>
<tr>
<td>Realized returns</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.823</td>
<td>0.791</td>
<td>0.724</td>
<td>0.783</td>
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<tr>
<td>SD</td>
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<td>4.943</td>
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<td>0.158</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
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<td>1.118</td>
<td>0.989</td>
<td>1.175</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SW model</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exponential AR</td>
<td>0.997</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero CAPM</td>
<td>0.969</td>
<td>0.972</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Zero-beta + AR</td>
<td>0.997</td>
<td>0.999</td>
<td>0.970</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: This table shows the average performance of minimum variance portfolios generated period by period using eq. (7). Expected returns are modeled using the SW model, the exponential autoregressive model with volatility, and the zero-beta CAPM with and without the first-order autoregressive term. The conditional covariance matrix is modeled using the BEKK parameterization, in which the squared lagged forecasting errors and cross-errors and the lagged conditional covariance matrix are used to generate the expected variances and covariances. Returns are monthly and in percentage form. Expected returns and variance on portfolios are the average fitted values.
FIGURE 2.— Mean-variance frontier and minimum-variance portfolios

Note: The figure shows the historical mean-variance frontier and the average performance of minimum variance portfolios generated period by period. Returns and standard deviations are monthly and in percentage form.

exponential AR allows also autocorrelations that vary with volatilities, the results show that the risk-return trade-off component can be important in practical applications based on the superior performance of the SW model.

Tangency portfolio with a risk-free asset

If there exists a risk-free asset, $R_f$, investors will hold a portfolio that consists of some combination of the risk-free asset and the tangency portfolio. The tangency portfolio is the optimal combination of the risky assets that is the same for all investors irrespective of their risk aversion. Mean-variance optimization implies the following portfolio allocation for the tangency portfolio without short-selling limitations:

$$W_{target} = \frac{H^{-1}(R - R_f)}{B - AR_f} \quad (8)$$

where
\[ A = l'H^{-1}1, \]

\[ B = l'H^{-1}R. \]

Eq. (8) utilizes a vector of expected returns, \( R \), which is likely to highlight differences between models. Thus, contrary to the previous section, the tangency portfolio solution shows both the direct effect of dynamic autocorrelation in portfolio choice. In the following analyses, we use the one-month Eurodollar rate observed at time \( t-1 \) for the risk-free rate calculations for time \( t \).

Although the solution with eq. (8) is unrestricted, following Kroner and Ng (1998), we consider a portfolio allocation decision with a no-shorting constraint. This is an illustrative case that portfolio managers face in practice.\(^{16}\) If short selling is restricted, eq. (8) cannot be used as such and numerical optimization is needed instead. However, since we analyze only two risky assets, it is easy to show that, given the solution with eq. (8), the optimal portfolio holding of the US stock market portfolio for the next period is

\[
 w^*_{US} = \begin{cases} 
 0 & \text{if } w_{US} < 0 \\
 w_{US} & \text{if } 0 \leq w_{US} \leq 1, \\
 1 & \text{if } w_{US} > 1 
\end{cases} 
\]

whereas the optimal holding of the Latin America stock portfolio is \( 1 - w^*_{US} \).

Table 5 shows the performance of different models. Again, it is apparent that taking autocorrelation in stock returns into consideration improves portfolio performance in terms of average return and risk-corrected average standardized return. Based on the realized figures, the zero-beta CAPM without the autoregressive term performs considerably worse than the models that include an autoregressive term. However, allowing time-varying autoregressive coefficients seems to be less important than in the previous section.

\(^{16}\) If there are no restrictions on short selling, optimal solutions with the estimated models occasionally suggest taking unrealistically large short positions. In most cases, margin limits and fund rules would prohibit a portfolio manager to take a huge short position in a particular stock market. Moreover, during the global financial crisis, authorities in numerous countries restricted short selling (Beber and Pagano, 2012).
The exponential AR with volatility, i.e. the third best performer, earns on average 0.163% percentage points per month more than the zero-beta CAPM portfolio. The resulting percentage differences in the two models in average return (19.5%) and standardized return (39%) are significant figures (a 2.00 percentage point difference per annum with a considerably lower standard deviation). The differences with other models are more modest. Interestingly, if the first-order autocorrelation is ignored, the fraction invested in an emerging stock market on average is greater than that invested in the US stock market portfolio.

### TABLE 5. Tangency portfolios with a risk-free asset and no short selling

<table>
<thead>
<tr>
<th></th>
<th>SW model</th>
<th>Exponential AR</th>
<th>Zero-beta CAPM</th>
<th>Zero-beta CAPM with AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction in US stock portfolio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.665</td>
<td>0.709</td>
<td>0.329</td>
<td>0.655</td>
</tr>
<tr>
<td>SD</td>
<td>0.392</td>
<td>0.382</td>
<td>0.289</td>
<td>0.391</td>
</tr>
<tr>
<td>Fraction in Latin America stock portfolio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.335</td>
<td>0.291</td>
<td>0.671</td>
<td>0.345</td>
</tr>
<tr>
<td>Realized returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.036</td>
<td>1.001</td>
<td>0.838</td>
<td>1.150</td>
</tr>
<tr>
<td>SD</td>
<td>6.160</td>
<td>6.000</td>
<td>6.994</td>
<td>6.248</td>
</tr>
<tr>
<td>Mean/SD</td>
<td>0.168</td>
<td>0.167</td>
<td>0.120</td>
<td>0.184</td>
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<tr>
<td>Expected returns</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.748</td>
<td>1.545</td>
<td>1.687</td>
<td>1.706</td>
</tr>
<tr>
<td>E[SD]</td>
<td>5.461</td>
<td>5.233</td>
<td>6.398</td>
<td>5.436</td>
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<tr>
<td>Correlations of realized returns</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>SW model</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exponential AR</td>
<td>0.884</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero CAPM</td>
<td>0.881</td>
<td>0.836</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Zero-beta + AR</td>
<td>0.952</td>
<td>0.868</td>
<td>0.904</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**Note:** This table shows the average performance of tangency portfolios, generated period by period, when the risk-free asset is available and short selling is forbidden. Expected returns are modeled using the SW model, the exponential autoregressive model with volatility, and the zero-beta CAPM with and without the first-order autoregressive term. The conditional covariance matrix is modeled using the BEKK parameterization. The risk-free rate is approximated using the one-month Eurodollar rate for the monthly risk-free rate calculations. Returns are monthly and in percentage form. Expected returns and variance on portfolios are the average fitted values.

The exponential AR with volatility, i.e. the third best performer, earns on average 0.163% percentage points per month more than the zero-beta CAPM portfolio. The resulting percentage differences in the two models in average return (19.5%) and standardized return (39%) are significant figures (a 2.00 percentage point difference per annum with a considerably lower standard deviation). The differences with other models are more modest. Interestingly, if the first-order autocorrelation is ignored, the fraction invested in an emerging stock market on average is greater than that invested in the US stock market portfolio.
FIGURE 3.—Cumulative performance

Note: The figure shows the cumulative in-sample performance of an original one-dollar investment in tangency portfolios generated period by period. The risk-free asset is available and short selling is forbidden. The allocations are done using the SW model, the exponential autoregressive model with volatility, and the zero-beta CAPM with and without the first-order autoregressive term.

Figure 3 plots the cumulative performance of a one-dollar original investment in tangency portfolios generated period by period using different models. The percentage point difference in the cumulative return over the 17-year sample period between the zero-beta CAPM without the autoregressive term and the SW model is approximately 200 percentage points. The difference between the zero-beta CAPM with and without the autoregressive term is approximately 350 percentage points. These are economically significant differences.

The correlation coefficients between the realized returns support the view that there is now a more significant difference between models than in the previous section. This seems reasonable as, contrary to the previous section, expected returns now directly affect the allocation decision through eq. (8). In general, comparison of the expected and realized returns and variances reveal that all models tend to overstate expectations.

When evaluating the previous results, it should be noted that this study does not consider the effect of transaction and information gathering costs on models’ performance. Gains from international
diversification are likely to be reduced, at least slightly, if these costs were included. Specifically, especially investments in emerging markets, can involve hard-to-quantify costs such as information gathering and processing costs. In the previous sections, the standard deviation of the mean investments in the US and Latin America stock portfolios is always lowest for the zero-beta CAPM without the autoregressive term. As Gerard and Wu (2006) mention, the standard deviation of the mean investment serves as a proxy for portfolio’s turnover. This indicates that the pure zero-beta CAPM, with least turnover, could perform better if transactions costs were included in the analysis. However, the differences in the standard deviations of mean investments compared to differences in model performance are small enough to conclude that the model rankings would stay essentially unchanged.

V. Conclusions

This paper studies the relevance of time-varying return autocorrelation in modeling expected returns and allocating funds between developed and emerging stock markets. The analysis is conducted using monthly US and Latin America aggregate stock market data. The SW model’s performance in explaining stock market returns is first evaluated against traditional asset-pricing models and an empirical model that allows for time-varying autocorrelation. The results are controlled against the influence of global pricing factors. Second, the direct and indirect effects of considering time-varying autocorrelation in international portfolio allocation are studied.

Consistent with previous studies, the study reveals that autocorrelation varies with volatility. The result from the SW model suggests that some investors’ demand for stocks is based on past price changes in both the US and Latin American market areas. In this case, both a conditional risk-return trade-off and autocorrelation can cause return predictability. Time-varying first-order autocorrelation in stock returns appears to be mostly positive in both stock markets – usually at a higher level in Latin America returns than in the US aggregate return. This finding agrees with the general view that autocorrelation is more important in emerging stock market returns. Contrary to previous authors, we control the results against the influence of global factors that may affect the findings regarding time-varying autocorrelation.
Global market returns and changes in the oil price as additional variables do not help forecasting returns in addition to a risk-return trade-off and a first-lag autoregressive component.

The SW model and the exponential autoregressive model fit the data considerably better than the conditional zero-beta CAPM. This implies that heterogeneous agent models and models that account for time-varying autocorrelation offer a more realistic description for stock returns than models assuming a single representative investor or ignore serial dependence in stock returns. However, differences in performance in explaining monthly returns between the SW model and alternative models with a first-order autoregressive term are modest.

Considering dynamic return autocorrelation can improve an investor’s portfolio allocation between emerging and developed stock markets. Using the SW model and its alternatives to model expected returns, we construct period by period both the global minimum variance portfolio and the tangency portfolio with the risk-free return and a shorting constraint. Based on the average performance of different models, accounting for time-varying autocorrelation can be economically important in portfolio optimization. Dynamic return autocorrelation can influence portfolio choice: both indirectly via the effect of forecasting errors on the conditional covariance matrix and directly through the expected returns on the stock markets. The most drastic difference in portfolios performance is not due to allowing autocorrelation to vary over time, but realizing that stock returns are autorecorrelated, especially in emerging stock markets. The portfolios constructed using the zero-beta CAPM without a first-order autoregressive term always significantly underperform in terms of risk and return characteristics.

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Prof. P. Theodossiou, Editor-in-Chief, June 2018

References


