Strategic exits in secondary venture capital markets

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Abstract

The market for secondary venture capital transactions allows investors to gain liquidity if the investee start-up corporation has not yet reached maturity for an IPO. At the same time, it creates divestment opportunities for badly performing ventures which should rather be abandoned. We analyze how opportunistic behavior and liquidity constraints of venture capital funds channel deal flow into the secondary venture capital market. The opportunistic behavior leads to strategic exits of seed financing venture capitalists. These exits enlarge the investors’ opportunity set of strategies and therefore, affect the deal terms with entrepreneurs. We show in this paper that two contracts are possible in a world with financially constrained venture capital investors, staged investments, and premature divestment opportunities. Both of them carry a particular disadvantage. With the first one, the venture capitalist never liquidates a project, even if it is a lemon, but rather engages in a secondary transaction. With the second contract, lemons get systematically abandoned but this can also happen to good quality ventures. Entrepreneurs need to consider these effects when they aim to maximize their benefit and need to trade off the contract parameters accordingly. Our model helps explaining the deal flow into the secondary venture capital market and allows several empirical predictions.


Keywords: Secondary Venture Capital Market, Entrepreneurial Finance, Venture Capital, Moral Hazards.

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1 Introduction

Investors were expected to lock their money into a venture capital (VC) fund for at least 10 years and entrepreneurs assumed VC funds to be strategic partners who accompany their venture for a long time. While both is true, in principle, a new phenomenon called the “secondary VC market” has established. In this market, individual claims of VCs\(^1\) and fund interests are traded. More recently, Fintechs, e.g. as described in Block et al. (2018) specializing on secondary market trading activities and “pre-IPO” (pre-Initial Public Offering) platforms have emerged and grow at a strong pace.\(^2\) Secondary transactions offer premature exit opportunities and provide liquidity to the VC market. Institutional investors of VC funds appreciate the rising activity. They even commit capital to specialized funds which act as acquirers. It seems, at first sight, that the possibility to gain liquidity can only benefit the VC asset class because it discharges an illiquidity premium. Contrarily, it is surprising that investors expect VC to become marketable while its economic virtue stems from tight long-term financing relationships with innovative ventures. The nature of these relationships is highly illiquid. It has further been pointed out, e.g. by Black and Gilson (1998), that VC exposure only generates appropriate returns if a venture reaches maturity and can be divested via an IPO, or a Trade Sale (divestment via the M&A market), alternatively. Hence, investors in VC assets are rather advised not to divest until an investee qualifies for such a “traditional” exit route to harvest higher returns.

Prior to the secondary market’s existence, VCs needed to bring good quality ventures to maturity and to abandon lemons timely. With the secondary market, VCs do now have the possibility to divest exposure prematurely as described in Cumming (2008) and Cumming and Johan (2008a and 2008b). However, aiming to generate a

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1. Note: We also use the abbreviation VC to denote “venture capitalist”.
higher return, a VC fund would probably not engage in a secondary transaction if the investee is successful and if there is no reason for him to sell his claim. Therefore, it is important to understand the drivers and characteristics of the deal flow in the secondary VC market. In principle, there can only be two reasons why a VC initiates a secondary transaction. The first reason is that he attempts to sell a claim in a promising venture because he faces a liquidity constraint. The second reason is that he tries to dispose of a lemon, pretending it to be a unicorn. This is possible because secondary transactions are subject to substantial information asymmetry between the deal partners. Following Admati and Pfeiderer (1994), a secondary transaction can result in a sale of a lemon if the insider is able to hide the true reason of his disposal. Hence, acquirers in secondary transactions cannot infer on the quality of the venture unless they know if the originator faces a liquidity constraint or not. Whatever the reason for a secondary transaction is, the new exit channel adds to a VC’s opportunity set of divestment strategies. It must therefore also affect the deal terms with an entrepreneur. No other paper has yet considered the new exit route in a formalized way focussing on the potential entrepreneurial finance contracts and this is our contribution.

We present a model which is based on the possibility that a seller in the secondary VC market intentionally benefits from the opacity of the true reason of his divestment. We consider an entrepreneur endowed with an innovative project who solicits outside financing. She is irreplaceable due to her specific knowledge and will manage the venture. Two capital contributions are required to back the investee through a seed and a development stage. At seed stage, a contract is negotiated with a VC and immediately after closing, the fund injects seed money into the young company. The entrepreneur determines her level of effort according to her incentives as defined in the contract. During the seed period, the VC supports and monitors the venture actively. This encourages the entrepreneur and thus, increases the likelihood of success. It also
allows the investor to gain private information about the quality of the venture so that he can determine its state of nature which is either good or bad. At the same time, he is privately informed whether he needs to exit the relationship. This requirement may result from fundraising pressure at the end of his fund’s lifetime even in case the venture is promising. A fund’s vintage is known by outsiders. Therefore, fundraising pressure is predictable and a common feature in the VC asset class, in principle. However, there remains the unpredictable duration of the development period of the venture which could shift a potential IPO or Trade Sale window beyond the fund’s lifecycle. Additionally, it is possible that the other portfolio investments of the same VC fund are not successful. Such information on fund performance is usually not available to the public but only to the fund’s investors. If current investors are not satisfied with the fund’s performance they might be reluctant to accept a lifetime-extension or to receive the securities held by the fund, but rather insist on its liquidation. In this case, the VC is required to exit by either winding up the venture, or by selling his claim to an outside investor in a secondary transaction. However, unless it has been winded up, the venture needs expansion capital. This must be provided by the respective acquirer if the venture goes through a secondary market transaction. If there is no need for the seed phase VC to exit and if the development of the venture is promising, then the VC continues backing the venture. Nevertheless, in this second situation, a new outside investor needs to step in for an expansion financing round. At the end of the development period, the venture is mature and can be divested via an IPO or a Trade Sale.

In this setting, we refer to the principle of strategic exits as defined in Faure-Grimaud and Gromb (2004) and analyze how a seed financing VC can use his insider information on the quality of a project to dispose of a claim in a bad venture in the secondary VC market. Our main hypothesis is that the acquirer does not know why a seed financ-
ing VC sells his stake instead of bringing the venture to an IPO or Trade Sale. His buying rationale is that the venture is a prospective unicorn but the seller has a financial constraint and is therefore required to liquidate assets. Our model yields several important results. In particular, we show that only two contracts are possible, corresponding to two semi-pooling equilibria. The first one supports the perception that the VC secondary market is a market for both, unicorns and lemons. Market participants dispose of good and bad ventures, contingent on several parameters discussed in our model. It is impossible to design a contract forcing the investor to disclose the true quality of a project in a secondary transaction. The seed investor hides this information and thus benefits due to the fact that he may truly be forced to exit. Next, we derive the second contract providing incentives which rule out strategic exits. Such a contract would force liquidation of bad projects instead of channeling them into the secondary VC market. However, it comes with a drawback which is the liquidation of eventually successful projects. This is certainly an unwarranted feature and we prove that it is therefore an imperfect solution to mitigate the opportunistic behavior of the insider. The comprehension of such opportunistic behavior is particularly important in the entrepreneurial finance context where information asymmetries are large and where evaluating investees is difficult for outsiders.

We contribute to the entrepreneurship literature and derive important implications for practice in several ways. Our model helps explaining the deal flow in the secondary VC market. We prove that it is impossible to design a contract such that the investor is forced to make optimal continuation decisions in a world with staged financing, premature exit opportunities, and potential liquidity constraints. There are only two contracts possible with each having particular disadvantages. The first one channels not only promising ventures but also lemons into the secondary VC market. The second contract is a lock-up contract which forces liquidations. It would dry out the secondary
market and would yield inefficient abandonments of favorable ventures, instead. Our model’s primitives correspond with entrepreneurial finance contract parameters. It is important for seed capital-raising entrepreneurs to trade off these terms to maximize their NPV based on one of the available contracts.

The organization of the paper is as follows: We first review adjacent literature. Then, we present the model and its assumptions. In the subsequent section we derive the first-best situation and the optimal contracts. Afterwards, we compare these contracts and discuss particular cases. Finally, we provide empirical predictions and conclude. All proofs are provided in the appendix.

2 Related literature

A comprehensive overview of VC contracting problems is provided in Cummind and Johan (2013). Our model builds on several papers from these strands of the entrepreneurship and economics literature. Schwienbacher (2013) compares startup financing by generalist and specialist investors. The specialist provides better support but incurs additional cost to fund later rounds. He investigates the mechanisms for entrepreneurs to trade off these two types of investors. De Bettignies and Brander (2007) also compare the two types of seed investors. They highlight the potential effect of a VC’s support encouraging entrepreneurial effort. However, enhanced entrepreneurial effort comes at some cost because she has to relinquish control and cash flow rights. Andrieu and Groh (2018) argue closely related to De Bettignies and Brander (2007) and to Schwienbacher (2013) but focus on the allocation of control rights to entrepreneurs or investors and their impact on venture continuation decisions. However, none of the contributions considers the existence of the secondary VC market and the effect of asymmetric information if an investor divests prematurely. The model in this paper
bridges this gap. It is similar to Andrieu and Groh (2012) who compare bank-affiliated and independent VC funds. Nevertheless, the investor’s exit in their model is triggered by an exogenous shock. Our paper elaborates on the endogeneity of a premature exit decision and its relevance for entrepreneurs and acquirers in the secondary VC market.

A related model is also proposed in Ueda (2004), where an entrepreneur benefits from investor support but may be expropriated of her idea. Our paper does not focus on expropriation but on opportunistic investor behavior. Elitzur and Gavious (2003) and Schwienbacher (2007) elaborate on how entrepreneurs trade off angel and VC financing. The angel and the seed stage VC in the Elitzur and Gavious (2003) model have similar characteristics as the investors in ours. However, they do not include the possibility for premature exits and the inevitable moral hazard. Schwienbacher (2007) focuses on an option to wait which can finally avoid an initial angel round. In our model, seed financing is deterministic to escape additional complexity. Hellmann and Thiele (2015) consider a world where an entrepreneur successively solicits angel and VC financing. They analyze the link between the angel and VC market and how the bargaining power among the two types of investors frames the sharing of rents. Renucci (2014) explicitly models bargaining power as being a consequence of the investor’s personal wealth. Cumming and Johan (2008) empirically demonstrate the influence of bargaining power on cash flow allocation and control rights. Our model does not elaborate on the allocation of bargaining power but is robust to alternative allocations.

A model close to ours is from Faure-Grimaud and Gromb (2004). They focus on the influence of an active majority shareholder’s exit on the stock price of a firm. In their model, a majority shareholder may be hit by a liquidity shock that forces him to exit before the effect of his value-adding activity is observed by the market. In our setting, exit is not an exogenous requirement but the result of a decision to liquidate assets. Cumming and Johan (2010) model the exit decision as a conflict between the
entrepreneur and the VC. Exit will be triggered if the VC’s projected marginal cost reach the level of the projected marginal value added through further VC backing. This approach allows a link with the divestment probability in our model. One could consider the rising liquidation pressure by a fund’s investors as part of the projected marginal cost. Nevertheless, our model is based on the liquidity constraint of a VC and not on potential conflicts with the entrepreneur. Aghion et al. (2004) also consider a framework in which an investor may exit from an investment before its maturity. In their model, the investor monitors the entrepreneur’s effort and coinvests with uninformed outsiders. As is it the case in our model, the investor may be forced to exit prematurely and the necessity is only privately observed. They show that on one hand it can be important to control the exit rights of an investor, but on the other hand this requires a compensation for the resulting illiquidity. Kandel et al. (2011) study VC funds organized as limited partnerships. They show that bad projects may be continued to camouflage them for the investors in the partnerships. At the same time, good projects may be liquidated because their duration is expected to be too long. These inefficiencies are socially not desirable. In this paper, we model this inefficiency in the particular case of an ex ante uncertainty about a premature exit of a VC fund. This ex ante uncertainty has not yet been formalized before. One reason is that this was not necessary prior to the rise of the secondary VC market because premature exits of VC exposure were difficult to achieve and therefore rather rare. Modeling the rationale for such divestments and their impact on start-up financing contracts therefore presents our important contribution to the literature.
3 The model

We consider a risk-neutral world with no discounting. An entrepreneur endowed with a non-bankable innovative project requires outside financing. As in Ueda (2004), we assume that investors compete à la Bertrand, thus being competitive while the entrepreneur has full bargaining power due to the uniqueness of her idea. The project requires two investments $I_0$ and $I_1$ at the beginning of the seed and development periods. We consider two financiers: a seed-stage and an expansion-stage VC fund. If the venture reaches maturity, it can be sold in an IPO or Trade Sale at the end of the development period. This provides exit proceeds $R$.

The entrepreneur exerts effort. We consider a unit cost of effort parameter $c_I$ to assess the cost of entrepreneurial effort. These costs are affected by the support and monitoring quality of the seed stage investor. A higher management support quality yields lower cost of effort, hence lower values for $c_I$. The level of entrepreneurial effort is $e$, such that $0 \leq e \leq 1$. The cost of effort is then defined by the term $c_I e^2$. The effort of the entrepreneur has an important role in our model. It determines the venture’s state of nature at the end of the seed phase which is either good or bad. We assume that the probability to reach the good state is equal to $e$. At this interim stage, the investor is privately informed about the project’s state of nature. If the state is good, the exit proceeds $R$ at the end of the expansion period are certain. If the state is bad there is still a chance to receive $R$ given probability $q$, or 0 with probability $(1 - q)$. The duration of the project is ex ante uncertain. It can turn out during the seed phase that it will take longer than anticipated to bring the venture to maturity. At the same time, the seed financing VC fund may arrive at its liquidation period. This is publicly observable, in principle, by the fund’s vintage. However, the fund’s overall investment portfolio might not be appealing to its investors - and this information is not publicly disclosed. The investors could therefore be reluctant to allow an extension
of the fund’s lifetime and insist on its liquidation. This information is also not available to the public. As a consequence, only the seed VC is privately informed whether he has to exit or not at interim state. We denote the probability that this happens by $p$. If the seed financing VC is required to exit he can either wind up the venture or sell his claim in a secondary transaction. If he is not forced to exit he has the same alternatives, plus the option to continue with the investee until its maturity. Winding up the venture yields liquidation proceeds $L$ and the VC holds a liquidation preference $L_I$. We assume these proceeds not to cover the initial investment, i.e. $L < I_0$. If the venture is continued, it requires expansion capital. If the seed VC has not exited the transaction, then a development-stage VC injects expansion money in a new financing round. If the venture went through a secondary transaction instead, then the acquirer injects the required capital.

The timeline of the game is presented in Figure 1:

We firstly focus on the opportunity set of strategies of the seed investor: If he is forced to exit, given probability $p$, he either sells his claim in a secondary transaction at interim state or winds up the venture. If he is not forced to exit, given probability $(1 - p)$, he can either liquidate, sell or continue backing the venture. All contract parameters are set $ex$ $ante$ and we assume that the entrepreneur has full bargaining

Figure 1 – Timeline of the model
power. She makes a take-it-or-leave-it offer to investors, as in Hellmann (2002). For simplicity, we assume that the parameters of the contract with the expansion investor are also negotiated ex ante.³

We summarize the strategy set in Table 1:

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Table 1: Opportunity set of strategies of the seed investor

If the venture goes through a secondary transaction the model requires that the acquirer injects $I_1$ right afterwards and that $qR > L$. This induces that there is no benefit from buying the company and liquidating it. Since the acquirer does not know the true reason for the secondary transaction nor about the quality of the venture, there is no way to set the contract parameters ex ante contingent on the nature of the event.

We define the following contract parameters:⁴

- $\delta$ denotes the seed VC’s cash flow claim which is sold in the secondary transaction.
- $\delta_1$ and $\delta_2$ denote the cash flow claims of the seed and expansion capital VCs, respectively, if the venture is not sold in a secondary transaction.
- $P_I$ denotes the price for the claim $\delta$ in the secondary transaction.
- $L$ denotes the liquidation proceeds of the venture. $L_I$ is the liquidation preference of the seed VC, the residual $(L - L_I)$ flows to the entrepreneur.

³ Given that the contract parameters are defined ex ante, they perfectly reflect the information set and respect the participation constraint of the expansion investor.

⁴ For simplicity, we assume that $\delta, \delta_1, \delta_2$ and $P_I$ are negotiated in the initial contract between the seed stage investor and the entrepreneur. This assumption is purely formal. The parameters are calculated according to the information set of the acquirer while buying the VC’s claim. A negotiation at interim stage would yield exactly the same values.
4 Optimal contracts

We subsequently derive the first-best situation and the optimal contracts corresponding to the game. We assume the behavior of utility maximizing agents if the entrepreneur has full bargaining power but address the model’s robustness with respect to a different bargaining power allocation further below.

In the first-best equilibrium, the revenue minus the cost of all agents is maximized. The net present value of the project is:

\[-I_0 + e(R - I_1) + (1 - e)L - c_l \frac{e^2}{2}.\]

To reach the first-best equilibrium, the entrepreneur should set her level of effort such that:

\[\max_e -I_0 + e(R - I_1) + (1 - e)L - c_l \frac{e^2}{2}.\]

Hence, the level of effort that maximizes the NPV is \(e^{FB} = \frac{R - I_1 - L}{c_l}.\)

The seed capital VC makes a continuation decision at the end of the seed period contingent on the requirement to exit or not. We successively investigate the two alternatives (exit or not) and determine which one of the two related contracts maximizes the entrepreneur’s profit, solving the optimization program.

Let us first assume that the investors of the seed VC force him to liquidate his fund and he therefore needs to exit. He either engages in a secondary transaction or winds up the venture. If liquidated, he faces a loss because at best, he recovers \(L < I_0\). Thus, he sells as long as \(P_I > L_I\) and only liquidates if \(L_I > P_I\).

Let us here assume \(P_I > L_I\) which simply becomes true by setting \(L_I = 0\). This way, the seed VC always initiates a secondary transaction.

Now we assume the second case, that the seed VC is not forced to exit and has

\[5. \text{It is assumed that } R - I_1 - L > 0 \text{ and } R - I_1 - L < c_l. \text{ Then, } 1 > \frac{R - I_1 - L}{c_l} > 0.\]
therefore three options: continuing, selling, or liquidating. However, liquidating is not appropriate because $P_I > L_I$. The VC may therefore decide to sell whatever the state of the venture, reaching a pooling equilibrium. Or, he may decide to stay whatever the state of the venture. Nevertheless, we show in the appendix that this is not an optimal strategy and should therefore be avoided. The intuition is that the seed investor should not continue if the venture is a lemon but would be better off with a secondary transaction or by liquidating it. An alternative strategy is to continue if the venture’s state is good or to sell it if a lemon. We prove in the appendix that i) this strategy dominates the pooling equilibrium (selling whatever state of nature) and that: ii) the pooling equilibrium replicates the other strategy (continuing in good states and selling in bad) if $p = 1$. As a result, a contract that we denote contract number one is possible, where the seed VC continues in good states if he is not forced to exit and sells via a secondary transactions in any other case.

If we now assume $L_I > P_I$, which is simply true by setting $P_I = 0$, then the seed investor is always better off by liquidating rather than selling. It is then also possible to find a second contract such that the investor has an incentive to liquidate.

Summarizing, there are two possible situations. In the first one, the project is never liquidated even if it is a lemon. The seed financing VC always sells it via a secondary transaction in a good or bad state of nature if he is required to exit. If he is not forced to exit, then he continues if it is successful or he initiates a secondary market transaction if it is a lemon. In the second situation, the seed VC always liquidates the venture except in the case where he does not need to exit and where the venture is successful. Both situations correspond to a semi-separating (or pooling) equilibrium and it is impossible to get a contract allowing to infer the true nature of the secondary transaction.

In the next section, we elaborate on the contracts’ optimal parameters corresponding to these two situations. Subsequently, we analyze the determinants of the entrepreneur’s
choice between the two contracts.

4.1 Contract inducing secondary transactions

We focus on contract number one where the investor always engages in a secondary transaction except if the venture is successful and he is not forced to exit. The resulting opportunity set of strategies is presented in Table 2:

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Table 2: Opportunity set of strategies of the seed investor with contract number one, ruling out liquidation

We subsequently derive the contract parameters which maximize the entrepreneur’s NPV. The index \( S \) (\( S \) for secondary transaction) denotes the parameters of contract one.

Under this contract the venture is never liquidated. Therefore, it only requires the following parameters: \( \delta_1 \) and \( \delta_2 \) (i.e. the cash flow rights of both investors if there is no secondary transaction), \( \delta \) (the cash flow claim of the expansion VC if the first enters into a secondary transaction because he is forced to exit), and \( P_S \) (the price for \( \delta \) in the secondary transaction).

For simplicity, we require that the entrepreneurial effort is inferior to the first-best effort. This is equivalent to the assumption that there is no problem from spending too much effort.

The entrepreneur maximizes her utility function:

\[
p(e(1 - \delta)R + (1 - e)(1 - \delta)qR) + (1 - p)(e(1 - \delta_1 - \delta_2)R + (1 - e)(1 - \delta)qR) - cI \frac{e^2}{2}
\]
\[ = (1 - e)(1 - \delta)qR + pe(1 - \delta)R + (1 - p)e(1 - \delta_1 - \delta_2)R - c_1 \frac{e^2}{2}. \]

The seed VC does not liquidate if:

\[ P_S > L_S. \]

We assume that this is true and verify this in the appendix.

The seed investor continues with the venture in its good state of nature, if he is not forced to exit and iff:

\[ \delta_1 R \geq P_S. \]

We assume that, even if the inequality is not strict, the seed investor always prefers continuing to selling.\(^6\)

He sells a lemon in a secondary transaction if he can continue backing the venture and iff:

\[ \delta_1 qR \leq P_S. \]

We assume that the inequality is strict.\(^7\)

To motivate his investment in the venture in the first place, he requires:

\[ [p + (1 - p)(1 - e)] P_S + (1 - p)e\delta_1 R - I_0 \geq 0. \]

Given the seed investor’s incentive constraints, he has no interest in deviating from the strategy. This strategy corresponds to a perfect bayesian equilibrium. The expansion investor’s participation constraint is determined by his beliefs over the seed investor’s

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\(^6\) We prove in the appendix that a pooling equilibrium strategy in which the investor always sells is not optimal, and therefore not possible.

\(^7\) We prove in the appendix that it is the case
behavior. It is therefore:

\[(1 - e)(\delta qR - I_1 - P_S) + ep(\delta R - I_1 - P_S) + (\delta_2 R - I_1)(1 - p)e \geq 0.\]

He can also infer the good quality of the venture in case the seed investor continues. The expansion investor requires:

\[\delta_2 R - I_1 \geq 0,\]

from which we deduce \(\delta_2 = \frac{I_1}{R}\) to maximize the entrepreneurial profit. If this was not true, the investor would make a loss and refuse to invest in the first place. We can therefore substitute the expansion investor’s participation constraint with the following term:

\[(1 - e)(\delta qR - I_1 - P_S) + ep(\delta R - I_1 - P_S) \geq 0.\]

Using \(\delta_2 = \frac{I_1}{R}\), the maximization program is:

\[
\max_{\delta,e,\delta_1,P_S} (1 - e)(1 - \delta)qR + pe(1 - \delta)R + (1 - p)e(R - \delta_1 R - I_1) - c_t \frac{e^2}{2},
\]

s.t. \(e \in \arg\max (1 - e)(1 - \delta)qR + pe(1 - \delta)R + (1 - p)e(R - \delta_1 R - I_1) - c_t \frac{e^2}{2},\)

\[\delta_1 R - P_S \geq 0,\]

\[P_S - \delta_1 qR > 0,\]

\[(1 - e)(\delta qR - I_1 - P_S) + ep(\delta R - I_1 - P_S) \geq 0,\]

\[[p + (1 - p)(1 - e)] P_S + (1 - p)e\delta_1 R - I_0 \geq 0.\]

The subsequent proposition defines the parameters of the optimal contract which rules
Proposition 1 The entrepreneur can raise capital under a first contract setting the following parameters:

\[ P_S = \delta_1 R = I_0, \]

\[ \delta_2 = \frac{I_1}{R}, \]

\[ \delta = \frac{c_I(\sqrt{\Delta_s} - \gamma)}{2(q-p)^2 R^2}, \]

with \( \gamma = qR + \frac{R(q-p)[(p-q)R + (1-p)(R-2I)]}{c_I}, \)

and \( \Delta_s = \gamma^2 - \frac{4}{c_I}(q-p)^2 R^2 \left[ -I + \frac{(p-q)R + (1-p)(R-I)}{c_I} [I(1-p)] \right]. \)

Three conditions for the exogenous parameters need to be satisfied for the contract to be feasible:

\[ \Delta_s \geq 0, \]

\[ \gamma \geq 0 \text{ or } -I + \frac{(p-q)R + (1-p)(R-I)}{c_I} [I(1-p)] \geq 0, \]

\[ (q-p)\delta R - (qR + I_0 - L) + p(qR + I) \leq 0. \]

We obtain an equilibrium which is not completely but partially separating. The expansion investor can only infer a good quality of the venture if the seed investor stays. If the seed VC divests in a secondary transaction the expansion investor does not receive any information about the venture’s state. The problem is that lemons are not liquidated, thus leading to inefficient continuations. The expansion investor needs to recover losses from lemons with profits from ventures in good states. The entrepreneurial effort is lower than first-best.

Several conditions need to be met for the contract to be possible. The chance \( q \) to still achieve an IPO or Trade Sale exit even in a bad state needs to be not too high.
compared to the probability $p$ that the seed investor is forced to exit. This should be the case at reasonable values of $q$ and $p$. Furthermore, the expansion investor requires a sufficiently high final payoff to be incentivized to invest.

### 4.2 Contract forcing liquidation

We focus now on the parameters of contract two where the seed investor always liquidates unless the venture is of good quality and he is not forced to exit. Index $L$ ($L$ for liquidation) denotes the parameters of this contract. To incentivize liquidation, we simply set $P_L = 0$, yielding $P_L < L_L$ and define the liquidation proceeds allocated to the entrepreneur to be $L_E = L - L_L$. Table 3 summarizes the seed investor’s opportunity set of strategies:

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Table 3: Opportunity set of strategies of the seed investor with contract two, incentivizing liquidation

Next, we derive the optimal contract parameters.

The entrepreneur’s profit is according to the function:

$$L_E(p + (1 - p)(1 - e)) + (1 - p)e(1 - \delta_1 - \delta_2)R - c_1 \frac{e^2}{2}.$$ 

The expansion investor only invests if the seed VC continues backing the venture. Given his beliefs, he requires:

$$\delta_2 R - I_1 \geq 0$$

We set $\delta_2 = \frac{I_1}{R}$ to maximize the entrepreneur’s profit. It is also necessary to have
$L_L > \delta_1 q R$ and $\delta_1 R > L_L$\footnote{We prove in the appendix that these inequalities are strict.} to meet the incentive constraints of the seed investor.

The seed investor’s participation constraint is:

$$L_L(p + (1 - e)(1 - p)) + e(1 - p)\delta_1 R - I_0 \geq 0.$$ 

A necessary condition for the viability of the liquidation process is $L_L \leq L$, and the cash flow rights of investors should be such that: $\delta_1 + \delta_2 \leq 1$ and $\delta \leq 1$.

The maximization program is hence, the following:

$$\max_{\delta, e, \delta_1, L_L} (p + (1 - p)(1 - e))(L - L_L) + (1 - p)e(1 - \delta_1 R - I_1) - c_I \frac{e^2}{2},$$

subject to

$$\delta_1 R - L_L \geq 0,$$

$$L_L - \delta_1 q R \geq 0,$$

$$(p + (1 - p)(1 - e))(L_L) + (1 - p)e\delta_1 R - I_0 \geq 0,$$

$$L - L_L \geq 0,$$

$$\delta_1 + \delta_2 \leq 1 \text{ and } \delta \leq 1.$$

The subsequent proposition defines the optimal contract parameters.

**Proposition 2** The entrepreneur may raise capital under a second contract with the following parameters:

$$L_L = L,$$

$$\delta_1 = \frac{R + L - I_1 - \sqrt{[R + L - I_1]^2 + 4\Phi}}{2R},$$
\[ \delta_2 = \frac{I_1}{R}. \]

A necessary condition is:

\[ R + L - I_1 \geq 2\sqrt{-\Phi}, \]

with \( \Phi = \frac{(L - I_0) c_I}{(1 - p)^2} - (R - I_1) L. \)

In comparison with the first contract, the second one avoids inefficient continuations. If the venture is a lemon it will be liquidated. However, the severe drawback of this contract is that the venture will also be liquidated if successful but if the seed VC is required to exit. It is impossible to design a contract without this shortcoming: the seed investor prefers not to reveal the information about the state of nature of the venture to make a higher profit. It is further impossible to deduce the true reason of the exit decision of the seed investor in a good state if he is required to exit.

With this contract, the entrepreneur receives zero proceeds in case the venture is liquidated. She is only compensated in good states and if the seed investor does not exit. This contract to be feasible requires certain characteristics. First, the probability \( p \) that the seed VC exits should not be too high, because liquidation of the venture creates a loss. The initial investment \( I_0 \) should not be too high compared to the liquidation proceeds \( L \). Otherwise the loss would be too material relative to the profit if the venture is not liquidated. The benefit from continuation \( R - I_1 \) also needs to be sufficiently large.

This optimal contract is similar to a lock up contract, in which secondary transactions do not happen. It forces the seed investor to continue until the project’s maturity. We note that with \( p = 0 \), the contract replicates the first-best equilibrium.
5 Determinants of the entrepreneur’s choice

In this section, we compare the determinants of the entrepreneur’s choice among the two possible contracts and discuss particular cases. Subsequently, we make empirical predictions.

5.1 Choice between the optimal contract inducing secondary transactions or forcing liquidations

The following proposition establishes the determinants of the entrepreneur’s choice under the assumption that it is not possible to know the true reason for the seed investor’s exit. The entrepreneur weighs the benefits and drawbacks of the two possible contracts and decides accordingly. Again, we denote by $S$ and $L$ respectively, the parameters of the first and second contract and note that the latter is only possible under the condition that $(R + L)$ is sufficiently large compared to $I_0$.

Proposition 3 The entrepreneur prefers the second contract under the following two conditions:

I. $(1 - p)^2(R - I_1 - L)^2 \geq 2c_I \varphi(e_S),$

II. $e_L \geq \frac{(1-p)(R-I_1-L)-\sqrt{(1-p)^2(R-I_1-L)^2-2c_I \varphi(e_S)}}{c_I},$

with $\varphi(e_S) = e_SR + (1 - e_S)qR - L - I_1 - c_I e_S^2.$

Several parameters determine the entrepreneur’s choice, accordingly. The amount of the final IPO or Trade Sale proceeds $R$, of the liquidation proceeds $L$ and the expansion money $I_1$ are most important. The entrepreneur needs to expect $R - I_1 - L$ sufficiently high for the second contract to be attractive. This amount is the wealth generated by the decision to continue net of its opportunity cost.
The second important parameter is the exit likelihood of the seed VC. The higher \( p \), the less appealing becomes contract two: If the seed investor is unable to continue (at probability \( p \)) the venture is always liquidated, independent of its state of nature. This creates a loss since \( L < I_0 \). In contrast, if there is no need to exit (at probability \( 1-p \)), an optimal continuation decision will be made. This means that the venture will be winded up in its bad and continued in its good state. We note that the function \( \varphi(e_S) \) determines this choice. It captures the value of the systematic continuation decision in the alternative contract number one net of the entrepreneur’s cost of effort. This value should not be too high to make contract number two attractive to the entrepreneur.

Furthermore, the unit cost of effort \( c_I \) affect the trade off. These cost are specific to the project and the entrepreneur. However, they can be reduced by the VC’s management support and monitoring activities. High cost make contract two more favorable because they lower entrepreneurial effort and therefore increase the likelihood for the venture to become a failure.

Finally, contract two becomes less attractive if the chance \( q \) to still reach the maturity for an IPO or Trade Sale in a bad state is high. This is intuitive because with \( q \) increases the propensity for a successful exit even if the entrepreneur shirks.

Summarizing, the entrepreneur needs to thoroughly consider combinations of all contract parameters to trade off the potential NPVs with the different contracts.

### 5.2 Choice with an investor who can provide seed and expansion capital

If we assume that the entrepreneur finds a “generalist” investor, e.g. a bank, as in Schwienbacher (2013) who can provide seed and expansion stage financing, then we can replicate the first-best equilibrium. If such a generalist entered into a secondary transaction any outside investor would infer that this generalist tries selling a lemon.
A generalist investor is therefore required to wind up bad and to continue only good ventures.

We can easily derive the contract with a generalist, accordingly. It is sufficient to set parameter \( p = 0 \) in contract two, with systematic liquidation. However, as also argued in Schwienbacher (2013), or in Andrieu and Groh (2012), a generalist investor may not be expected to be as good as a VC fund in providing management support to a venture. In our model, selecting a generalist investor would therefore result in a higher \( c \) parameter, i.e., a higher unit cost of effort. We subsequently compare the two detected contracts with a contract with a generalist investor.

5.2.1 Comparison with the contract forcing liquidation

Let \( c_G \) be the exogenous unit cost of effort, corresponding to the productivity of entrepreneurial effort with a generalist investor.

The entrepreneur has to decide between a seed-financing VC or a generalist investor. We assume that the contract forcing liquidation is feasible and more attractive, according to the conditions of proposition 3. The subsequent proposition defines the determinants of the entrepreneur’s choice between the two investors:

**Proposition 4** The entrepreneur prefers the VC compared to a generalist if the two following conditions are true:

1. \( (1 - p)^2(R - I_1 - L)^2 \geq 2c_I \varphi_G(e_G) \),

2. \( e_L \geq \frac{(1 - p)(R - I_1 - L) - \sqrt{(1 - p)^2(R - I_1 - L)^2 - 2c_I \varphi_G(e_G)}}{c_G} \),

with \( \varphi_G(e_G) = e_G(R - I_1 - L) - I_0 - c_G \frac{e_G}{2} \).

The contract with the VC is advantageous under several conditions. The most important is the resulting level of entrepreneurial effort. Large differences of the quality of the management support will favor the VC because they increase the likelihood of
success and the project’s NPV. Additionally, the probability $p$ that the VC has to exit should not be too high. The higher it is, the more likely good projects will be liquidated. The impact of the value of the continuation decision in good states $R - I_1 - L$ is ambiguous.

We note that, if a generalist provided an equal level of support quality as a VC, then the generalist perfectly replicates the first-best equilibrium and would always be the best alternative. However, if this was the case in practice, then VCs would most probably not exist.

5.2.2 Comparison with the contract inducing secondary transactions

We assume now that the contract forcing liquidation is not possible or not the appropriate alternative, i.e., the conditions defined in proposition 3 are not simultaneously verified. This yields systematic continuation of the venture independent of its state of nature. We compare this outcome and that of a contract with a generalist investor in the following proposition:

**Proposition 5** The entrepreneur prefers the VC compared to a generalist if the following two conditions are true:

I. $(R)^2(1-q)^2 \geq 2c_I \varphi_G(e_G)$,

II. $e_S \geq \frac{R(1-q) - \sqrt{(R^2(1-q)^2 - 2c_I \varphi_G(e_G))}}{e_I}$,

with $\varphi_G(e_G) = e_G(R - I_1 - L) - qR + I_1 - c_G \frac{e_G}{2}$.

In this particular case, the VC generates an inefficiency because he never liquidates but rather engages in secondary transactions. We note that the premature exit probability $p$ drops out of the equation. Again, several characteristics determine the choice of the entrepreneur between the two investors. The first one is the unit cost of effort with the generalist $c_G$. The lower the cost, the more attractive is the generalist for the
entrepreneur. The unit cost of effort also influence the level of entrepreneurial effort and hence, the likelihood for a successful project. We emphasize again that if the support quality of the generalist was as good as that of the VC, then the generalist dominates, given that all other exogenous model parameters are equal.

The second important characteristic is the level of the liquidation proceeds $L$. Higher proceeds render the generalist more attractive. In contrast, higher success chances even in a bad state of nature $q$ favor the VC because this increases expected pay-offs while shirking.

Summarizing, the entrepreneur selects between the two investors by weighing the benefits resulting from a better support quality of the VC against the appropriate liquidation decisions with a generalist.

### 5.3 Allocation of bargaining power and renegotiation

Contingent on market dynamics, venture particularities and characteristics of the entrepreneur and the VC, the negotiation power could switch from the entrepreneur to investor. Therefore, we consider the alternative allocation of the bargaining power to the seed investor. In this case, the investor is no longer in perfect competition but recovers the project’s NPV. The liquidation preference, the purchase price in the secondary transaction and his cash flow claim are then set by him. However, a shift of the bargaining power does not change the general trade off previously established:

If the seed investor is required to exit, then he has the alternative to liquidate or to enter into a secondary transaction. With the entrepreneur holding the bargaining power, she receives 0 liquidation proceeds to boost her effort. With the investor holding the bargaining power, he will fix the liquidation proceeds allocation in exactly the same way to boost effort and to maximize the project value. If the liquidation proceeds are not too high, contract number one can be replicated: the VC always engages in a secondary
transaction to avoid liquidation losses except he is not forced to divest and the venture reaches a good state of nature in the seed period. If the liquidation proceeds are high he can set \textit{ex ante} the contract parameters to favor liquidation, exactly as described in contract number two. Summarizing, a change in the allocation of the bargaining power does not affect the model’s two equilibria.

It could further be possible that the contract partners ask for renegotiation at interim state to avoid inefficient continuation decisions. However, renegotiations can only happen if all parties agree, i.e. if they are better off by signing a new contract. Since the VC already has the full liquidation preference there is no possiblity to put him in a better position, which could avoid channeling a lemon into the secondary VC market with contract one. Similarly, to renegotiate contract number two would require to find a solution which prevents the VC to abandon good projects if he is forced to exit. In this case the price of the secondary transaction needs to be above the venture’s liquidation proceeds. However, this price cannot be defined contingent on the exit requirement of the VC because this is private information. Setting the secondary market transaction price \textit{ex ante} above the VC’s liquidation preference would rule out any liquidation and thus, replicate the first contract. Renegotiation is therefore also impossible with the second contract.

5.4 Empirical predictions and discussion

Our model allows to derive several empirical predictions. The setting reveals that there are only two contracts possible with a seed-financing VC in a world where the funds are exposed to certain liquidity constraints because they are structured as limited partnerships. Both contracts have particular drawbacks. The first contract yields inefficient continuation of bad projects while the second one induces systematic liquidation even of good projects. We should therefore expect a characteristic divestment behavior
according to the levels of financial constraints of VC funds. Funds which are more prone to liquidate assets for their investors, e.g. because they are structured as limited partnerships, should have a lower propensity to abandon ventures but to rather engage in the secondary VC market. In contrast, investors with a large pool of resources or VC funds structured in a different way (e.g. corporate, affiliated, quoted, open end, government, or simply large funds) are expected to be less financially constrained and hence, less active in the secondary VC market. They should have a higher propensity to either abandon transactions, even if liquidation proceeds are low, or to keep non-successful exposure on their books without taking any action. Empirical research on the secondary VC market could test this prediction analyzing the impact of affiliations and fund size of the vendors. It seems further important to address the role of expected liquidation values, e.g. via the ratio of tangible to total assets as in Rajan and Zingales (1995), with respect to exit-timing and exit-channel of VC transactions. Our model predicts that transactions with low expected liquidation proceeds should be more often channeled into the secondary VC market.

In addition to the expectations of the impact of affiliation, structure or size on a fund’s activity in the secondary market, our theory also contributes to the understanding of the behavior of individual seed financing VCs. In a perfect world, an unconstrained VC would not sell a good project at the interim state but would develop it to maturity and would liquidate it timely if it is bad. However, if a fund is constrained, then our model proposes that he engages in a secondary transaction independent of the project quality. Hence, the secondary VC market is fueled by financial constraints and deal flow includes promising ventures and lemons at the same time. Nevertheless, these constraints are not equally distributed over the lifetime of a VC fund but rise with its age. We therefore hypothesize that older vintage year funds engage in the secondary market more frequently. Additionally, we hypothesize that a fund’s performance in-
duces secondary market activity. If fund investors are satisfied with the performance they may allow extensions of its lifetime or the transfer of securities held by the fund. The VC is therefore not required to liquidate assets. Hence, we predict that successful VC funds are less constrained in the sense of our model and therefore less active players in the secondary VC market.

6 Conclusion

The secondary VC market is a new phenomenon which mitigates liquidity concerns of investors who have an interest to turn the long-term illiquid assets typically held by VC funds into marketable securities. While this desire is comprehensible and allows to discharge an illiquidity premium from the required return on VC investments, it is in contrast with the nature of the long-term and tight relationship between a VC and an entrepreneurial venture. It is also questionable if investors are able to capture appropriate returns if exposure is divested prematurely in the secondary VC market. Nevertheless, this market grows at a strong pace and its existence adds to the opportunity set of strategies of seed phase investors. This must affect the deal terms with entrepreneurs and we aim to contribute to understanding the effects in this paper. Therefore, we model the conflict between an eventually financially constraint VC and the funding needs of a start-up venture. An entrepreneur endowed with an innovative project solicits seed and expansion capital from two respective VCs. It is possible, however, that the seed VC cannot stay until the project’s maturity but needs to liquidate his fund’s assets. As a consequence, he either triggers his liquidation preference or sells his claim in a secondary transaction. If sold, the related uncertainty corresponds to a moral hazard situation for the buyer because the requirement of the VC to divest is not observable by outsiders. This introduces the possibility that the insider takes
advantage by making strategic exits via secondary transactions. Our model shows that only two situations are possible in this world.

In the first one, the project is never liquidated even if it is a failure. The VC rather engages in a secondary deal and sells a lemon. The VC would also sell if the project is promising but if he is asked by his investors to liquidate the current fund. Accordingly, this contract explains the flow of good and bad deals into the secondary VC market.

The second situation is similar to a lock up contract. The venture is systematically liquidated, even in a good state of nature, if the investor is forced to exit. Good projects would only be continued if the seed investor is able to stay.

The resulting two optimal contracts have strengths and drawbacks and the entrepreneur determines her contract choice by trading-off several important parameters to maximize her NPV. Our model proves that an ideal early stage investor would be able to provide a high quality management support and would have a sufficient capacity to provide subsequent financing. The model is renegotiation proof and robust with respect to different allocations of bargaining power. We predict a different characteristic divestment behavior in the secondary market of VC funds contingent on their fund life cycle, structure, affiliation, and fund performance. These are interesting avenues for subsequent empirical research.
7 Appendix

Proof of proposition 1

The entrepreneur sets her level of effort so that she maximizes her utility function:

\[(1 - e)(1 - \delta)qR + pe(1 - \delta)R + (1 - p)e(1 - \delta_1 - \delta_2)R - cI \frac{e^2}{2}.\]

Her level of effort is then:

\[e^* = \frac{(p - q)(1 - \delta)R + (1 - p)(1 - \delta_1 - \delta_2)R}{cI}.\]

A necessary incentive constraint for the seed investor to continue backing the venture in a good state of nature and if he has the possibility to continue is:

\[\delta_1 R \geq P_s \geq \delta_1 qR.\]

We investigate which one of the two constraints is binding to maximize the entrepreneur’s revenue (previously we have supposed \(P_s > L\)).

Let us assume that \(\delta_1 R > P_s\). The seed investor’s participation constraint is the following:

\[P_s(p + (1 - p)(1 - e)) + e\delta_1 R = I_0.\]

Let us increase \(P_s\) in \(\varepsilon > 0\). Then, \(\delta_1\) reduces in \(\frac{(p + (1 - p)(1 - e))\varepsilon}{\varepsilon R}\), and \(e^*\) increases (we have supposed that \(e^*\) is inferior to first-best effort). The entrepreneurial profits increase. To maximize these profits, it is necessary to set \(P_s = \delta_1 R\). In the same way, we have \(\delta_2 = \frac{I_1}{R}\). By replacing in the equation, we find \(P_s = I_0 > L\). The condition \(\delta_1 qR \geq P_s\) is impossible since \(\delta_1 qR = qP_s\) and \(q < 1\). The investor cannot continue when the state is bad.

By transforming the seed investor’s participation constraint, we find \(P_s \geq I_0\). As \(P_s = \delta_1 R, P_s = I_0\) to maximize the entrepreneurial profits.
Let us define $I = I_1 + I_0$. We check that $\delta_1 R + \delta_2 R = I \leq R$.

We now investigate the optimal value of $\delta^*$. We know that:

$$(1 - e)(\delta q R - I_1 - P_S) + e p(\delta R - I_1 - P_S) = \delta q R - I - e(\delta R(q - p) - I(1 - p)) \geq 0,$$

$$e^* = \frac{(p - q)(1 - \delta) R + (1 - p)(R - I)}{c_I}.$$

The previous inequation may be rewritten as follows:

$$\delta q R - I - \frac{(p - q)(1 - \delta) R + (1 - p)(R - I)}{c_I} [\delta R(q - p) - I(1 - p)] \geq 0.$$

We note that:

$$\frac{(p - q)(1 - \delta) R + (1 - p)(R - I)}{c_I} = \frac{[(p - q)R + (1 - p)(R - I)](I(1 - p)) + \delta \left[ \frac{R(q - p)((p - q)R + (1 - p)(R - I) + (p - q)R I(1 - p))}{c_I} \right] + \delta^2 \left[ \frac{(p - q)^2 R^2}{c_I} \right]}{\frac{qR + R(q - p)((p - q)R + (1 - p)(R - I) + (p - q)R I(1 - p))}{c_I} + \left[ -I + \frac{(p - q)R + (1 - p)(R - I)}{c_I} [I(1 - p)] \right]}.$$

The characteristics of the polynomial function $a\delta^2 + b\delta + c$ are:

$$a = \frac{(p - q)^2 R^2}{c_I} \geq 0,$$

$$b = qR + \frac{R(q - p)((p - q)R + (1 - p)(R - 2I))}{c_I},$$

$$c = -I + \frac{(p - q)R + (1 - p)(R - I)}{c_I} [I(1 - p)].$$

A first necessary condition for the existence of $\delta$ is:

$$\Delta_s = b^2 - 4ac \geq 0.$$

If $c \geq 0$, the condition is always true.
The two roots corresponding to the polynomial function are \(-\frac{b+\sqrt{\Delta}}{2a}\). The lowest is \(-\frac{b-\sqrt{\Delta}}{2a}\).

We must find a solution \(\delta\) such that \(0 \leq \delta \leq 1\). \(\delta\) should be as low as possible to maximize the entrepreneurial profits. \(a > 0\), then it is impossible that \(\delta^* = 0\) (otherwise the expansion investor would make a losses). It is impossible that \(\delta^* = \min(0; \max(0, -\frac{b-\sqrt{\Delta}}{2a}))\).

The only possible solution is \(\delta^* = -\frac{b+\sqrt{\Delta}}{2a}\).

A necessary condition is:

\[ b \leq \sqrt{\Delta} \iff b \leq 0 \text{ or } -4ac \leq 0 \iff b \leq 0 \text{ or } c \geq 0. \]

Proof of initial assumptions:

We first check that \(e^* = \frac{(p-q)(1-\delta)R+(1-p)(R-I)}{c_I}\) is inferior to first-best effort \(e^{FB} = \frac{R-I_1-L}{c_I}\).

\[ e^* - e^{FB} \leq 0 \iff (p-q)(1-\delta)R + (1-p)(R-I) \leq R - I_1 - L. \]

We check that the seed investor cannot decide to sell when he has to exit and always continues otherwise (e.g., in bad states), i.e. \(\delta_1qR \geq P_S\).

To maximize the entrepreneur’s profits, we would have \(\delta_1qR = P_S\), then \(\delta_1 = \frac{P_S}{qR}\).

The seed investor makes zero profit on average. His participation constraint would be:

\[ pP_S + (1-p)(e\delta_1R + (1-e)\delta_1qR) - I_0 \geq 0 \]

\[ \iff P_S \geq \frac{I_0}{\frac{e(1-p)}{q} + (1-e) + ep}. \]

The latter condition is binding to maximize the entrepreneur’s profits. Given that \(\frac{e(1-p)}{q} + (1-e) + ep > 1 \iff 1-q > 0\), \(P_S < I_0\).
In this case, the price would be below the price with the contract defined in proposition 1 and the condition is strict. □

We now prove that the pooling equilibrium strategy (selling whatever happens) is not dominant.

To derive the optimal contract parameters with this equilibrium, one has simply to set \( p = 1 \) and calculate the parameters according to proposition 1.

In this case, if the seed investor sells even if he can continue and the venture's state is good, then he obtains \( P_S = I_0 \). In good states, if he stayed he would have an equilibrium \( \delta_1 R = I_0 \). Allocating \( \varepsilon > 0 \) to him creates an incentive to do so.

The entrepreneur is better off if the seed investor continues backing the venture. Suppose the levels of effort are the same in both situations. The difference of the NPV between the final contract and the pooling equilibrium contract of the two situations would be \((1 - \varepsilon)p(L - qR + I_1)\). It is positive since \( qR > L \). Since the effort of the entrepreneur is lower than first-best, it is boosted by a higher NPV and the entrepreneur rejects the pooling equilibrium contract. □

**Proof of proposition 2:**

We investigate the contract under which the seed investor liquidate the venture.

The entrepreneur’s profit is: \( L_E(p + (1 - p)(1 - \varepsilon)) + (1 - p)\varepsilon(1 - \delta_1 - \delta_2)R - c_I \varepsilon^2 \).

The entrepreneur would set her effort such that:

\[
e^* = \frac{-(1 - p)(L_E) + (1 - p)(1 - \delta_1 - \delta_2)R}{c_I} = \frac{(1 - p) [(1 - \delta_1 - \delta_2)R - L_E]}{c_I}.
\]

A necessary condition for \( e^* \geq 0 \) is \( L_E \leq (1 - \delta_1 - \delta_2)R \).

The expansion investor’s participation constraint is:

\[
\delta_2 R - I_1 \geq 0,
\]
in which we set \( \delta_2 = \frac{I_1}{R} \) to maximize the entrepreneur’s profits.

Other conditions are \( L_L \geq \delta_1 qR \) and \( \delta_1 R \geq L_L \). We assume here that even if the inequalities are strict, the seed investor always prefers liquidating or staying. We then prove that they are strict.

The seed investor’s participation constraint is:

\[
L_L (p + (1 - e)(1 - p)) + e(1 - p)\delta_1 R - I_0 \geq 0.
\]

Let us assume that \( \delta_1 R = L_L \). It is equivalent to allocating the maximal amount of liquidation proceeds to the seed investor. It would result in the fact that the seed investor’s participation constraint cannot be satisfied since this would imply \( L_L - I_0 \geq 0 \) (which contradicts one initial assumption). We then have \( \delta_1 R > L_L \).

Consequently, the only constraints relative to \( L_L \) are: \( L_L \leq L \) and \( L_L \geq \delta_1 qR \).

Let us assume \( L_L < L \). Let us increase \( L_L \) in \( \varepsilon > 0 \). Then, in the seed investor’s participation constraint, one can note that \( \delta_1 \) diminishes in \( \frac{(p + (1 - e)(1 - p))\varepsilon}{e(1 - p)R} \). In addition, \( e^* \) increases (which is decreasing with \( \delta_1 \)). We now assume that the entrepreneurial effort is lower than first best (which needs to be checked afterwards). Hence, the profit of the entrepreneur increases. Consequently, \( L_L = L \) and \( L_E = 0 \).

We will check afterwards that \( L > \delta_1 qR \).

The seed investor’s participation constraint now becomes:

\[
L_L (p + (1 - e)(1 - p)) + e(1 - p)\delta_1 R - I_0 \geq 0.
\]

As \( e^* = \frac{(1-p)(R-\delta_1 R - I_1)}{c_L} \), we have:

\[
L - I_0 + e(1 - p)(-L + \delta_1 R) \geq 0,
\]
\[ L - I_0 + \frac{(1-p)^2(R - \delta_1 R - I_1)(-L + \delta_1 R)}{c_I} \geq 0, \]
\[ \delta_1^2 \frac{R^2(1-p)^2}{c_I} + \delta_1 \frac{(1-p)^2 LR + R(1-p)^2(R - I_1)}{c_I} + L - I_0 - \frac{(1-p)^2(R - I_1)L}{c_I} \geq 0 \]
\[ \iff \delta_1^2 [-R^2] + \delta_1 [LR + R(R - I_1)] + \frac{(L - I_0)c_I}{(1-p)^2} - (R - I_1)L \geq 0. \]

The characteristics of the polynomial function are:

\[ a = -R^2 < 0, \]
\[ b = LR + R(R - I_1) > 0, \]
\[ c = \Phi = \frac{(L - I_0)c_I}{(1-p)^2} - (R - I_1)L \geq 0 < 0. \]

The discriminant is \( \Delta = b^2 - 4ac. \) It is positive iff: \( b^2 \geq 4ac, \) otherwise the function is negative and the contract is not feasible. We set it as a condition of the contract.

This condition can be rewritten as \( b \geq 2\sqrt{-R^2\Phi} \iff R + L - I_1 \geq 2\sqrt{-\Phi}. \)

The first polynomial’s root is \( \frac{-b - \sqrt{\Delta}}{2a} > 0 \) as also the second one \( \frac{-b + \sqrt{\Delta}}{2a} > 0 \) which is the lowest.

To maximize her profits, the entrepreneur will set:

\[ \delta_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{[LR + R(R - I_1)] + \sqrt{[LR + R(R - I_1)]^2 + 4R^2\Phi}}{-2R^2} \]
\[ \iff \delta_1 = \frac{LR + R(R - I_1) - \sqrt{[LR + R(R - I_1)]^2 + 4\Phi}}{2R} , \]

with \( \Phi = \frac{(L - I_0)c_I}{(1-p)^2} - (R - I_1)L. \)

Proof of initial assumptions:
We first check that $\delta_1 q R < L_L = L$. One sufficient condition would be $\frac{q(R-I_1)-L}{2} < 0$, which is true since $q R - I_1 - L < 0$ by assumption.

We then check that $\delta_1 R + \delta_2 R \leq R$. It is equivalent to:

$$R - I_1 - \frac{R - I_1 + L - \sqrt{[LR + R(R - I_1)]^2 + 4\Phi}}{2} \geq 0$$

$$\iff R - I_1 - L + \frac{\sqrt{[LR + R(R - I_1)]^2 + 4\Phi}}{2} \geq 0.$$

The latter is always true since $R - I_1 - L \geq 0$.

Last, we check that $e^*$ is inferior to first-best effort:

$$e^* = \frac{(1-p)(R - \delta_1 R - I_1)}{c_I} \leq \frac{R - I_1 - L}{c_I}$$

$$\iff (1-p)(R - \delta_1 R - I_1) \leq R - I_1 - L$$

$$\iff (-p)(R - \delta_1 R - I_1) - \delta_1 R + L \leq 0.$$

It is always true since $\delta_1 R > L$.

Proof of proposition 3

We compare the global NPV obtained with the two contracts. We denote with $S$ the parameters of the first contract inducing secondary transactions and with $L$ the parameters of the second contract forcing liquidations. The contract with liquidation generates higher NPV iff:

$$pL + (1-p)(1-e_L)L + e_L(1-p)(R-I_1)-I_0-c_I\frac{e_L^2}{2} - \left[ e_S R + (1 - e_S)q R - I_0 - I_1 - c_I \frac{e_S^2}{2} \right] \geq 0$$

$$\iff e_L^2(-\frac{c_I}{2}) + e_L(1-p)(R - I_1 - L) - \left[ e_S R + (1 - e_S)q R - L - I_1 - c_I \frac{e_S^2}{2} \right] \geq 0.$$
The polynomial function $ax^2 + bx + c$ is such that $a < 0$ ; $b > 0$ et $c > 0$ (one may easily note in $c$ the NPV of the systematic continuation decision of the project).

The function is positive if the discriminant $\Delta$ is positive. A first necessary condition is:

$$(1 - p)^2(R - I_1 - L)^2 \geq 2cL \left[e_S R + (1 - e_S)qR - L - I_1 - cI \frac{e_G^2}{2}\right].$$

A second necessary condition is: $-\frac{b + \sqrt{\Delta}}{2a} \leq e_L \leq -\frac{b - \sqrt{\Delta}}{2a}$. As $-\frac{b}{2a} = \frac{(1-p)(R-I_1-L)}{cL}$ and $e_L = \frac{(1-p)[R(1-\delta_1)-I_1-L+L_S]}{cL}$, we have:

$$-\frac{b}{2a} - e_L = \frac{(1-p) [R(\delta_1) - L_S]}{cL} > 0,$$

because the seed investor prefers continuing rather than liquidating in a good state with this contract. Consequently, the condition $e_L \leq -\frac{b - \sqrt{\Delta}}{2a}$ is always true, as $-\frac{\sqrt{\Delta}}{2a} = \frac{\sqrt{\Delta}}{cL} > 0$. The other condition with respect to $e_L$ is not always true.

**Proof of proposition 4**

The contract with the seed investor with liquidation is more attractive than the contract with a generalist investor iff:

$$pL + (1-p)(1-e_L)L + e_L(1-p)(R-I_1) - I_0 - cL \frac{e_G^2}{2} - [e_G(R - I_1) + (1 - e_G)L - I_0 - cG \frac{e_G^2}{2}] \geq 0$$

$$\iff e_L^2(-\frac{e_G^2}{2}) + e_L(1-p)(R - I_1 - L) - [e_G(R - I_1 - L) - I_0 - cG \frac{e_G^2}{2}] \geq 0.$$  

The polynomial function is $ax^2 + bx + c$ with $a < 0$ ; $b > 0$.

The value of $c$ is:

$$c = e_G(R - I_1 - L) - I_0 - cG \frac{e_G^2}{2}$$

$$\iff c = \frac{(R - I_1 - L)^2 + (R - I_1 - L)\sqrt{(R - I_1 - L)^2 + 4c_G(L - I_0)}}{2c_G} - I_0$$

37
\[-\frac{(R-I_1-L)^2+(R-I_1-L)\sqrt{(R-I_1-L)^2+4c_G(L-I_0)}+2c_G(L-I_0)}{4c_G}\]

\[\iff c = \frac{(R-I_1-L)^2+(R-I_1-L)\sqrt{(R-I_1-L)^2+4c_G(L-I_0)}-I_0-L}{2}.\]

We note that \(c\) has an undetermined sign.

For the polynomial function to be positive, it must have a positive discriminant \(\Delta\).

A first necessary condition is:

\[(1-p)^2(R-I_1-L)^2 \geq 2c_I \left[e_G(R-I_1-L)-I_0-e_G \frac{e_G^2}{2}\right].\]

A second necessary condition is:

\[-\frac{b+\sqrt{\Delta}}{2a} \leq e_L \leq \frac{-b-\sqrt{\Delta}}{2a}.\]

We know that \(\frac{-b}{2a} = \frac{(1-p)(R-I_1-L)}{c_I}\) and \(e_L = (1-p)[R(\delta_1)-I_L+L_L]/c_I\). We have:

\[-\frac{b}{2a} - e_L = \frac{(1-p)[R(\delta_1)-L_L]}{c_I} > 0,
\]

because the seed investor prefers continuing rather than liquidating in a good state if he is able to continue backing the venture with this contract. Consequently, the condition \(e_L \leq \frac{-b-\sqrt{\Delta}}{2a}\) is always true, as \(\frac{\sqrt{\Delta}}{2a} = \frac{\sqrt{\Delta}}{c_I} > 0\). The second condition with respect to \(e_L\) is not always true.

Summarizing, the two conditions that need to be simultaneously satisfied are:

I. \((1-p)^2(R-I_1-L)^2 \geq 2c_I \left[e_G(R-I_1-L)-I_0-e_G \frac{e_G^2}{2}\right],\)

II. \(e_L \geq \frac{(1-p)(R-I_1-L)-\sqrt{(1-p)^2(R-I_1-L)^2-2c_I \left[e_G(R-I_1-L)-I_0-e_G \frac{e_G^2}{2}\right]}}{c_I}.

Proof of proposition 5
The contract with the seed investor with liquidation is more attractive than the contract with a generalist investor iff:

\[ e_L R + (1 - e_L) q R - I_0 - I_1 - c_I \frac{e_L^2}{2} - \left[ e_G (R - I_1) + (1 - e_G) L - I_0 - c_G \frac{e_G^2}{2} \right] \geq 0 \]

\[ \iff e_G^2 \left( -\frac{c_I}{2} \right) + e_L (R(1 - q)) + q R - I_1 - \left[ e_G (R - I_1 - L) - c_G \frac{e_G^2}{2} \right] \geq 0. \]

The polynomial function is \( ax^2 + bx + c \) with \( a < 0 \); \( b > 0 \).

For the polynomial function to be positive, it must have a positive discriminant \( \Delta \).

A first necessary condition is:

\[ (R)^2 (1 - q)^2 \geq 2 c_I \left[ e_G (R - I_1 - L) - q R + I_1 - c_G \frac{e_G^2}{2} \right]. \]

A second necessary condition is: \( -\frac{b + \sqrt{\Delta}}{2a} \leq e_C \leq -\frac{b - \sqrt{\Delta}}{2a} \). As \( -\frac{b}{2a} = \frac{R(1-q)}{c_I} \) and \( e_C = \frac{(p-q)(1-\delta)R+(1-p)(1-\delta_1-\delta_2)R}{c_I} \), we have:

\[ -\frac{b}{2a} - e_C = \frac{\delta R(p + q) - (1 - p)(I_0 + I_1)}{c_I} > 0, \]

as \( p \delta R - I_0 - I_1 > 0 \) (otherwise the expansion investor makes a loss if the seed investor exits and this would violate his participation constraint). The condition \( e_L \leq -\frac{b - \sqrt{\Delta}}{2a} \) is always true, as \( -\frac{\sqrt{\Delta}}{2a} = \frac{\sqrt{\Delta}}{c_I} > 0 \). The other condition with respect to \( e_L \) is not always true.
References


